Honors Algebra 2 – Summer Review Packet

Welcome to Honors Algebra 2! We look forward to having you in class next year.

In order to keep your Algebra 1 math skills sharp and to help prepare you for Honors Algebra 2 next year, please complete this Algebra 1 review of basic skills. These topics are covered in Chapter 1 of our textbook. By completing this packet, we will be able to do a very quick review of Chapter 1 and get started with one of the major topics of Algebra 2, linear equations and functions.

Directions: Please complete the *ODD* problems at the end of each section in the summer review packet. Use the examples and explanations provided to help you solve the problems. If you need further assistance, please consult khanacademy.org, YouTube or other Internet resources. All problems should be completed **WITHOUT a calculator.** This review will count as a graded assignment and will be collected. **All problems should be completed on a separate sheet of paper with the work shown and answers circled.** The Chapter 1 Test will be a non-calculator test based on the material covered in this packet. This test will be given within the first week of school.

Thank you and we look forward to seeing you in September!

Mrs. Kelly and Mr. Stout

Subsets of the Real Numbers:

Whole numbers: 0,1,2,3,.......

Integers: ....,-3,-2-1,0,1,2,3,................

Rational numbers: numbers such as $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{-4}{1}$ (or -4) that can be written as the ratio of two integers. When written as decimals, rational numbers terminate or repeat. *For example:* $\frac{3}{4} = 0.75$ *and* $\frac{1}{3} = 0.333....$

Irrational numbers: Real numbers that are not rational, such as $\sqrt{2}$ and π. When written as decimals, irrational numbers neither terminate nor repeat.

Real Number properties

	Addition	Multiplication
COMMUTATIVE	$a + b = b + a$	$ab = ba$
ASSOCIATIVE	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
IDENTITY	$a + 0 = a, 0 + a = a$	$a * 1 = a, 1 * a = a$
INVERSE	$a + (-a) = 0$	$a * \frac{1}{a} = 1, a \neq 0$
DISTRIBUTIVE	$a(b+c) = ab + ac$ (for both addition and multiplication)	

Example 1: Ordering numbers

Write the numbers from least to greatest

a. $-\frac{4}{3}$, 2.7, *and* $\sqrt{2}$

Answer: $-\frac{4}{3}$, $\sqrt{2}$ *and* 2.7

Example 2: Identify Properties of Real Numbers

a. $(3+9) + 8 = 3 + (9+8)$ **Answer:** Associative property

b. $14 * 1 = 14$ **Answer:** Identity property

Example 3: Operations with real numbers

a. The difference of 7 and -10 is:

$$7 - (-10) = 7 + 10$$

$$= 17$$

b. The quotient of -24 and $\frac{1}{3}$ is:

$$-\frac{24}{\frac{1}{3}} = -24 * 3$$

$$= -72$$

Practice B

For use with pages 3–10

Plot the numbers on a number line. Decide which number is greater and use the symbol < or > to show the relationship.

1. $-7, -10$

2. $\frac{3}{4}, \frac{7}{4}$

3. $\frac{10}{3}, \frac{17}{4}$

4. $-\frac{25}{2}, -\frac{14}{5}$

5. $0.8, -0.9$

6. $\sqrt{3}, 2.5$

7. $-\sqrt{2}, -\frac{2}{5}$

8. $-3.2, -4.1$

9. $-\frac{8}{3}, -\frac{7}{5}$

Write the numbers in increasing order.

10. $2, -\frac{3}{2}, -4, 1, \frac{1}{2}$

11. $\frac{7}{2}, \frac{3}{4}, -\frac{1}{3}, 0$

12. $\frac{1}{5}, 2.1, -\sqrt{7}, -3$

13. $8, \frac{15}{2}, -2.9, -\sqrt{10}$

14. $\sqrt{2}, 3, -\frac{5}{2}, -\sqrt{3}, 1$

15. $\frac{7}{3}, -\sqrt{5}, -6, \frac{13}{4}, -\frac{1}{2}$

Identify the property shown.

16. $(-6)(1) = -6$

17. $(3 + 1) + 2 = (1 + 3) + 2$

18. $7 + [2 + (-2)] = 7 + 0$

19. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

20. $a + (b + c) = (a + b) + c$

21. $a + 0 = a$

Select and perform an operation to answer the question.

22. What is the sum of -8 and 3?

23. What is the sum of -12 and -8?

24. What is the difference of 4 and 8?

25. What is the difference of -5 and -2?

26. What is the product of -4 and -6?

27. What is the product of -9 and -3?

28. What is the quotient of -36 and 9?

29. What is the quotient of -12 and $-\frac{2}{3}$?

30. *Filing Cabinet* A cabinet has 4 drawers. Each drawer is 13 inches tall. How tall is the cabinet?

31. *Touchdown* A football team scored 18 of their 27 points from touchdowns. If a touchdown is worth 6 points, how many touchdowns did the team score?

32. *Eating Pizza* Eight friends buy 4 pizzas. Each pizza is cut into 6 pieces. Each person eats the same number of pieces. How many pieces does each person eat?

33. *Playing Golf* The following table shows how many strokes over or under par Susan shot when she played nine holes of golf on Saturday. How far over or under par was her final score?

Hole	1	2	3	4	5	6	7	8	9
Score	-2	1	0	2	1	-1	-1	0	-2

1.2 Algebraic Expressions and Models

The expression 2^5 represents the number that you obtain when 2 is used as a factor 5 times.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

The number 2 is the base, the number 5 is the exponent, and the expression 2^5 is a power.

Example 1: Evaluating Powers

a. $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$

b. $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

Order of Operations (PEMDAS)

1. First, do operations that occur within grouping symbols (parenthesis).
2. Next, evaluate powers (exponents).
3. Then, do multiplications and divisions from left to right.
4. Finally, do additions and subtractions from left to right.

Example 2: Using Order of Operations

a. $-4 + 2(-2 + 5)^2 = -4 + 2(3)^2$
$$= -4 + 2(9)$$
$$= -4 + 18$$
$$= 14$$

A variable is a letter used to represent one or more numbers.

Example 3: Evaluating an Algebraic Expression

a. **Evaluate:** $-3x^2 - 5x + 7$ when $x = -2$
$$= -3(-2)^2 - 5(-2) + 7$$
$$= -3(4) + 10 + 7$$
$$= -12 + 10 + 7$$
$$= 5$$

Example 4: Write and evaluate a real life model

   a. You have $50 to buy movies that cost $15 each write an expression that shows how much money you have left after buying n movies.

$$50 - 15n$$

   b. Evaluate if you buy 2 movies.

$$50 - 15(2) = \$20$$

Example 5: Simplifying by Combining Like Terms

   a. $7x + 4x = 11x$

   b. $3n^2 + n - n^2 = (3n^2 - n^2) + n$
$$= 2n^2 + n$$

   c. $2(x+1) - 3(x-4) = 2x + 2 - 3x + 12$
$$= -x + 14$$

Example 6: Using a Real Life Model

   a. You want to buy either a CD or a cassette as a gift for each of 10 people. CD's cost $13 each and cassettes cost $8 each. Write an expression for the total amount you must spend.

     Price per CD × Number of CD's + Price per cassette × Number of cassettes

     13n + 8(10 – n) = 13n + 80 – 8n

$$= 5n + 80$$

   b. Then evaluate the expression when 4 of the people get CD's.

$$= 5n + 80$$
$$= 5(4) + 80$$
$$= 20 + 80$$
$$= \$100$$

Write the expression using exponents.

1. $a \cdot a \cdot a \cdot a \cdot a$

2. $(-9)(-9)(-9)(-9)$

3. $(-x)(-x)(-x)$

4. $(2y \cdot 2y \cdot 2y) + 7$

5. $(4b \cdot 4b) + (2a \cdot 2a)$

6. 8 to the nth power

Evaluate the expression.

7. $(-3)^4$

8. -2^6

9. $-(-2)^5$

10. $3 \cdot (2 + 1) - 4$

11. $14 \div (7 - 5) + 1$

12. $-1 + (3 + 2)^2$

13. $(5 - 2)^3 - 3 \cdot 4$

14. $2 \cdot (3 + 1)^2$

15. $(5 - 2)^3 \div 9 - 6$

Evaluate the expression for the given value of x.

16. $x(2 + x)$ when $x = -5$

17. $3x - 0.5(x + 1)$ when $x = 3$

18. $x^2 + 3x$ when $x = 4$

19. $2x \div (1 - x)$ when $x = 5$

20. $(25x - 3) \div 8$ when $x = 3$

21. $6 - x^3 + x$ when $x = -2$

Evaluate the expression for the given values of x and y.

22. $2x + y^3$ when $x = 3$ and $y = -2$
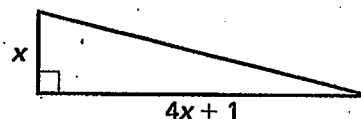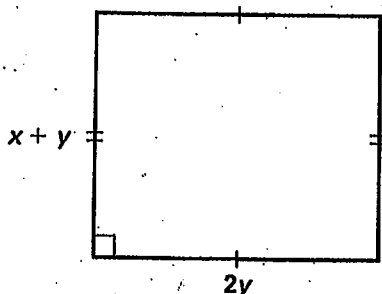
23. $(2y)^3 - 5x$ when $x = 2$ and $y = 1$

24. $\dfrac{3x + y}{2x - 1}$ when $x = 3$ and $y = 1$

25. $\dfrac{(y - 2)^3}{2x + y}$ when $x = -1$ and $y = 4$

Write an expression for the area of the figure. Then evaluate the expression for the given value(s) of the variable(s).

26. $x = 2$ and $y = 3$

27. $x = 6$

28. *Photography Studio*   A photography studio advertises a session with a sitting fee of $8.95 per person. The standard package of pictures costs $29.95. Write an expression that gives the total cost of a session plus the purchase of one standard package. Evaluate the expression if a family of four purchases this package.

29. *Books*   You want to buy either a paperback or hard covered book as a gift for 5 friends. Paperbacks cost $6.95 each and hard covered books cost $24.99 each. Write an expression for the total amount you must spend. Evaluate the expression if 3 of your friends get a paperback.

30. *Weekly Earnings*   For 1980 through 1990, the average weekly earnings (in dollars) for workers in the United States can be modeled by $E = 14.5t + 270$, where t is the number of years since 1980. Approximate the average weekly earnings in 1980 and 1990.

Lesson 1.2

1.3   Solving Linear Equations

Example 1: Solving an equation with a variable on one side

a. **Solve:** $\frac{3}{7}x + 9 = 15$

$$\frac{3}{7}x = 6$$

$$\left(\frac{7}{3}\right)\frac{3}{7}x = 6\left(\frac{7}{3}\right) \quad \leftarrow \text{(Multiply both sides of the equation by the reciprocal)}$$

$$x = 14$$

Example 2: Solving an equation with a variable on both sides

a. **Solve:** 5x + 11 = 7x - 9

$$-2x = -20$$
$$x = 10$$

Example 3: Using the Distributive Property

a. **Solve:** 4 (3x – 5) = -2 (-x + 8) – 6x

$$12x - 20 = 2x - 16 - 6x$$
$$12x - 20 = -4x - 16$$
$$16x = 4$$
$$x = \frac{4}{16}$$
$$x = \frac{1}{4}$$

Example 4: Solving an equation with fractions

a. **Solve:** $\frac{1}{3}x + \frac{1}{4} = x - \frac{1}{6}$

$$(12)\left(\frac{1}{3}x + \frac{1}{4}\right) = \left(x - \frac{1}{6}\right)(12) \quad \leftarrow \text{(Multiply both sides of the equation by the LCD)}$$

$$4x + 3 = 12x - 2$$

$$-8x = -5$$

$$x = \frac{5}{8}$$

Example 5: Writing and Using a Linear Equation

a. A real estate broker's base salary is $18,000. She earns a 4% commission on total sales. How much must she sell to earn $55,000 total?

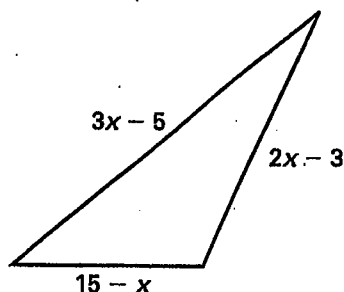Answer: $55{,}000 = 18{,}000 + 0.04x$

$37{,}000 = 0.04x$

$\$925{,}000 = x$
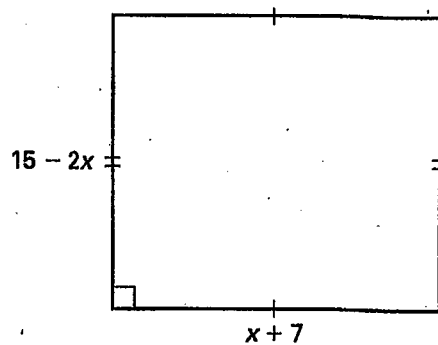
Solve the equation. Check your solution.

1. $x - 8 = 12$

2. $2x + 3 = 7$

3. $5x - 2 = 13$

4. $6 - x = 4$

5. $9 - 3x = -3$

6. $8x + 3 = 5$

7. $3x - 5 = 9$

8. $x + 4 = 2x + 9$

9. $3x - 1 = x + 4$

10. $4 + 5x = x - 8$

11. $\frac{1}{2}x + 6 = -4$

12. $\frac{2}{3}x - 1 = x + 7$

13. $-(x + 1) = 2(3x - 1)$

14. $3(x - 2) = 5(4 + x)$

15. $2(7 - x) = 6(1 + 2x)$

16. $3(x + 4) = 3(8 - 2x)$

17. $\frac{1}{2}(4x + 10) = 5 - 3x$

18. $\frac{1}{3}x + 1 = -\frac{1}{3}x - 8$

19. $\frac{3}{2}(x - 5) = 7$

20. $\frac{1}{4}x + 2 = 3 - \frac{3}{4}x$

21. $5(2x - 2) = 4 - 2x$

Find the dimensions of the figure.

22. The perimeter of the figure is 35 feet.

$3x - 5$

$2x - 3$

$15 - x$

23. The perimeter of the figure is 38 feet.

$15 - 2x$

$x + 7$

24. *Sales Tax* The state sales tax in Pennsylvania is 0.06 (or 6%). If your total bill at the music store included $1.32 in tax, how much did the merchandise cost?

25. *Movie Tickets* A ticket to the movies costs $7. You have $21. How many tickets can you buy?

26. *Weekly Pay* You have a summer job that pays $5.60 an hour. You get $8.40 an hour for overtime (anything over 40 hours). How many hours of overtime must you work to earn $287?

27. *Plumbing Bill* The bill from your plumber was $134. The cost for labor was $32 per hour. The cost for materials was $46. How many hours did the plumber work?

28. *Travel Time* You want to visit your aunt who lives 255 miles away. The interstate is 10 miles from your house and once you get off the interstate, you must travel 14 miles more to get to your aunt's house. If you drive 55 miles per hour on the interstate, how many hours will you travel on the interstate?

29. *Babysitting Rate* You charge $2 plus $.50 per child for every hour you babysit. You earn $3.50 an hour when you watch the Crandell children. How many children are in this family?

1.4   Rewriting equations and formulas

Example 1: Rewriting an equation with one variable

a. **Solve:** $7x - 3y = 8$ for y

$$-3y = -7x + 8$$

$$y = \frac{7}{3}x - \frac{8}{3}$$

Example 2: Calculating the value of a variable

a. Given the equation $x + xy = 1$; solve the equation for y first, then find the value of y when $x = -1$.

Solve the equation for y first: $x + xy = 1$

$$xy = -x + 1$$

$$y = \frac{-x+1}{x}$$

Then find y when $x = -1$: $y = \frac{-(-1)+1}{-1}$

$$y = -2$$

Common Formulas:

	Formula	Variables
Distance	$d = rt$	d=distance, r=rate, t=time
Simple Interest	$I = Prt$	I=interest, P=principal, r=rate, t=time
Temperature	$F = \frac{9}{5}C + 32$	F=degrees Fahrenheit, C=degrees Celsius
Area of a Triangle	$A = \frac{1}{2}bh$	A=area, b=base, h=height
Area of Rectangle	$A = lw$	A=area, l=length, w=width
Perimeter of Rectangle	$P = 2l + 2w$	P=perimeter, l=length, w=width
Area of Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	A=area, b_1= one base, b_2=other base, h=height
Area of Circle	$A = 2\pi r$	A=area, r=radius
Circumference of Circle	$C = 2\pi r$	C=circumference, r=radius

Example 3: Rewriting a common formula

a.   The formula for the perimeter of a rectangle is P=2l+2w. Solve for w.

$$P=2l+2w$$
$$P-2l=2w$$
$$\frac{P-2l}{2}=w$$

Find the value of y for the given value of x by first substituting the value of x into the equation and solving for y.

1. $3x - 7y = 8$; $x = -2$ **2.** $x = 12 - xy$; $x = 4$ **3.** $5x + 2y = 8$; $x = -1$

4. $\frac{4}{5}x = \frac{3}{2}y - 4$; $x = 10$ **5.** $\frac{2}{3}x + \frac{1}{2}y = 6$; $x = 6$ **6.** $2x + 3y = 1$; $x = -1$

Find the value of y for the given value of x by first solving for y and then substituting the value of x into the equation.

7. $6x - 9y = 9$; $x = -3$ **8.** $-3x + 7 = 2y + 3$; $x = 4$ **9.** $8x + 3y = 10$; $x = 4$

10. $2 + xy = 5x$; $x = -1$ **11.** $\frac{3}{4}x + \frac{4}{7}y = -6$; $x = 8$ **12.** $\frac{3}{5}x - \frac{2}{9}y = -13$; $x = 15$

Solve the formula for the indicated variable.

13. *Height of an Equilateral Triangle*

Solve for s: $h = \frac{\sqrt{3}}{2}s$

14. *Perimeter of an Equilateral Triangle*

Solve for s: $P = 3s$

15. *Volume of a Right Circular Cone*

Solve for h: $V = \frac{\pi r^2 h}{3}$

16. *Celsius to Fahrenheit*

Solve for C: $F = \frac{9}{5}C + 32$

17. *Area of a Trapezoid*

Solve for h: $A = \frac{h}{2}(b_1 + b_2)$

18. *Area of a Trapezoid*

Solve for b_2: $A = \frac{h}{2}(b_1 + b_2)$

19. *Lateral Surface Area of a Right Circular Cylinder*

Solve for r: $S = 2\pi rh$

20. *Volume of a Right Circular Cylinder*

Solve for h: $V = \pi r^2 h$

Solve the formula for the indicated variable. Then evaluate the rewritten formula for the given value(s). (Include units of measure in the answer.)

21. *Perimeter of a Square:* $P = 4s$
Solve for s.
Find s when $P = 44$ cm.

22. *Area of a Rectangle:* $A = lw$
Solve for l.
Find l when $A = 24$ ft^2 and $w = 8$ ft.

Hot Air Balloons In 1794, the French Army sent soldiers up in hot air balloons to observe enemy troop movements. One such balloon, the L'Entrepenant, had a volume of $\frac{256\pi}{3}$ cubic meters.

23. Solve the formula for the volume of a sphere $\left(V = \frac{4}{3}\pi r^3\right)$ for r^3. Then use this formula to calculate the radius of the L'Entrepenant balloon.

24. What was the diameter of the L'Entrepenant balloon?

25. Use the formula for surface area of a sphere $(S = 4\pi r^2)$ to approximate the surface area of the L'Entrepenant balloon.

1.5 Problem solving using algebraic models

Example 1: Writing and using a formula

a. The Bullet Train runs between the Japanese cities of Osaka and Fukuoka, a distance of 550 kilometers. When it makes no stops, it takes 2 hours and 15 minutes to make the trip. What is the average speed of the Bullet Train?

$$D = rt$$

$$550 = r(2.25)$$

$$244 = r$$

Answer: 244 kilometers per hour

Example 2: Writing and using a simple model

a. A water saving faucet has a flow rate of at most 9.6 cubic inches per second. To test whether your faucet meets this standard, you time how long it takes the faucet to fill a 470 cubic inch pot, obtaining a time of 35 seconds. Find your faucet's flow rate. Does it meet the standard for water conservation?

Volume of pot = Flow rate of faucet × time to fill the pot

$$470 = r (35)$$
$$13.4 = r$$

Answer: The flow rate is 13.4 cubic inches per second, which does not meet the standard of 9.6 cubic inches per second.

Example 3: Writing and using a model

a. You own a lawn care business. You want to know how much money you spend on gasoline to travel to out of town clients. In a typical week you drive 600 miles and use 40 gallons of gasoline. Gasoline costs $1.25 per gallon, and your truck's fuel efficiency is 21 miles per gallon on the highway and 13 miles per gallon in town.

Total miles = (highway fuel efficiency × amount of highway gasoline) + (local fuel efficiency × amount of local gasoline)

$$600 = 21x + 13(40-x)$$

$$600 = 21x + 520 - 13x$$

$$600 = 8x + 520$$

$$80 = 8x$$

$$10 = x$$

Answer: In a typical week you use 10 gallons of gasoline to travel to out of town clients. The cost of the gasoline is (10 gallons) ($1.25 per gallon) = $12.50

Practice B

For use with pages 33–40

Land Speed Record   **In Exercises 1–4, use the following information.**
The land speed record was broken in 1997 by a British car called the Thrust SSC.
The Thrust SSC traveled at a rate of 763 miles per hour. This was accomplished by
using a jet engine. How long would it take the Thrust SSC to travel 100 miles?
Use the following verbal model.

$$\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$$

   1. Assign labels to the parts of the verbal model.

   2. Use the labels to translate the verbal model into an algebraic model.

   3. Solve the algebraic model.

   4. Answer the question.

New Carpeting   **In Exercises 5–9, use the following information.** You
just added a family room to your home. You have budgeted $450 for carpeting. If
you need 30 square yards of carpeting, how much can you spend per square yard?

   5. Write a verbal model.

   6. Assign labels to the parts of the verbal model.

   7. Use the labels to translate the verbal model into an algebraic model.

   **8.** Solve the algebraic model.

   9. Answer the question.

Sharing the Driving   **In Exercises 10–14, use the following information.**
You and a friend share the driving on a 300 mile trip. Your friend drives for
3 hours at an average speed of 52 miles per hour. How fast must you drive for the
remainder of the trip if you want to reach your hotel in 3 more hours?

   10. Write a verbal model.

   11. Assign labels to the parts of the verbal model.

   12. Use the labels to translate the verbal model into an algebraic model.

   13. Solve the algebraic model.

   14. Answer the question.

Time Management   **In Exercises 15–19, use the following information.**
You need to do an experiment at home for your science class and write a lab report
on your findings. The experiment involves trials that take 5 minutes each to perform.
You want to watch a basketball game that starts in $1\frac{1}{2}$ hours. If it takes about 30 min-
utes to write the lab report, how many trials can you perform before the game starts?

   15. Write a verbal model.

   16. Assign labels to the parts of the verbal model.

   17. Use the labels to translate the verbal model into an algebraic model.

   18. Solve the algebraic model.

   19. Answer the question.

1.6   Solving Linear Inequalities

Example 1: Solving an equality with a variable on one side

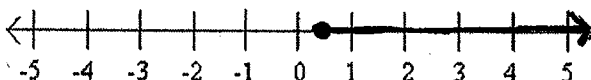a. Solve: $5y - 8 < 12$

$5y < 20$

$y < 4$
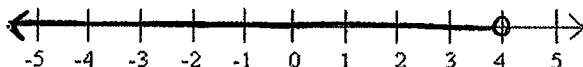
Example 2: Solving an inequality with a variable on both sides

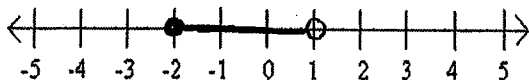a.   Solve: $2x + 1 \leq 6x - 1$

$-4x \leq -2$

$x \geq \frac{1}{2}$

A compound inequality is two simple inequalities joined by "and" or "or".
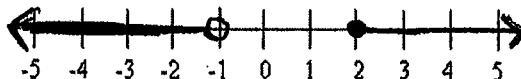
For example:

$-2 \leq x < 1$ $x < -1 \; or \; x \geq 2$

(all real numbers that are greater than   (all real numbers that are less than -1

or equal to -2 *and* less than 1)   *or* greater than or equal to 2)

Example 3: Solving an "and" compound inequality

    a. **Solve:** $-2 \leq 3x - 8 \leq 10$

                $6 \leq 3x \leq 18$

                $2 \leq x \leq 6$

Example 4: Solving an "or" compound inequality

    a. **Solve:** $2x + 3 < 5 \; or \; 4x - 7 > 9$

             $2x < 2 \; or \; 4x > 16$

             $x < 1 \; or \; x > 4$
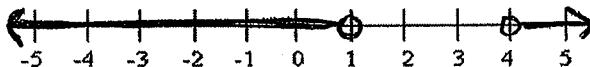
Example 5: Using a compound Inequality

    a. You are a state patrol officer who is assigned to work traffic enforcement on a highway. The posted minimum speed on the highway is 45 miles per hour and the posted maximum speed is 65 miles per hour. You need to detect vehicles that are traveling outside the posted speed limits. Write these conditions as a compound inequality.

        Let m represent the vehicle speeds in miles per hour. The speeds that you need to detect are given by: $m < 45 \; or \; m > 65$

1.6

Practice B

For use with pages 41–47

Graph the solution of the inequality.

1. $x < 4$

2. $x > -3$

3. $x \le -1$

4. $x \ge 7$

5. $3 < x < 5$

6. $x \le -4$ or $x \ge -1$

Solve the inequality.

7. $x + 8 < 14$

8. $-11x > 77$

9. $2x - 1 > -5$

10. $3x + 2 < 8$

11. $5x - 8 \ge -3$

12. $\frac{1}{2}x + 4 \le 7$

13. $-x + 5 \ge 6$

14. $4 - 2x \le 0$

15. $-3x + 5 > -1$

16. $7 - 9x < 12$

17. $-5x + 1 \ge 1$

18. $3x - 1 \le 2x + 2$

19. $-2 < 2x - 5 < 3$

20. $-4 < 2 - x < 6$

21. $x - 4 \le 2$ or $x - 4 \ge 12$

22. $x - 1 < -3$ or $x - 1 > 3$

23. $3 \le \frac{1}{2}x - 1 \le 5$

24. $2(x - 3) < 8$

Solve the inequality. Then graph the solution.

25. $\frac{2}{3}x - 5 > 1$

26. $6 + 3x \le 5$

27. $3 - x > 2$

28. $7 - \frac{3}{2}x \ge 6$

29. $1 - 2x > x + 10$

30. $2(4 - x) \ge 6$

31. *Extreme Points* The northernmost point of the United States is Point Barrow, Alaska. It lies on the 71°23′ latitude line. The southernmost point of the United States is Ka Lae, Hawaii. It lies on the 18°55′ latitude line. Write an inequality that represents the various latitudes of locations in the United States.

32. *Exam Grades* The grades for a course are based on 5 exams and 1 final. All six of the tests are worth 100 points. In order to receive an A in the course, you must earn at least 540 points. Your grades on the 5 exams are as follows: 87, 95, 92, 81, and 89. Write an inequality that represents the various grades you can earn on the final and still get an A. Solve the inequality.

33. *Speed Limit* The speed limit on a certain stretch of highway is 65 miles per hour. Write an inequality that represents the distances you can travel if you obey the speed limit for 2 hours. Solve the inequality.

34. *January Temperatures* The highest January temperature in the United States was 98° F in Laredo, Texas in 1954. The lowest January temperature in the United States was −80° F in Prospect Creek, Alaska in 1971. Write an inequality that represents the various temperatures in the United States during January.

35. *Bird Eggs* The largest egg laid by any bird is that of the ostrich. An ostrich egg can reach 8 inches in length. The smallest egg is that of the vervain hummingbird. Its eggs are approximately 0.4 inch in length. Write an inequality that represents the various lengths of bird eggs.

1.7 Solving Absolute Value Equations and Inequalities

Example 1: Solving an Absolute Value Equation

a. Solve: $|2x - 5| = 9$

$$-9 = 2x - 5 = 9$$
$$-4 = \quad 2x \quad = 14$$
$$x = -2 \text{ and } x = 7$$

Example 2: Solving an Inequality of the form $|ax + b| < c$

a. Solve: $|2x + 7| < 11$

$$-11 < 2x + 7 < 11$$
$$-18 < \quad 2x \quad < 4$$
$$-9 < \quad x \quad < 2$$

Example 3: Solving an Inequality of the form $|ax + b| \geq c$

a. Solve: $|3x - 2| \geq 8$

$$3x - 2 \geq 8 \quad \text{or} \quad 3x - 2 \leq -8$$
$$3x \geq 10 \quad \text{or} \quad 3x \leq -6$$
$$x \geq \frac{10}{3} \quad \text{or} \quad x \leq -2$$

Example 4: Writing a model for tolerance

a. A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for "20 ounce" boxes.

| Actual weight – Ideal weight | ≤ Tolerance

Actual weight = x   Ideal weight = 20   Tolerance = 0.75

$$|x - 20| \leq 0.75$$
$$-0.75 \leq x - 20 \leq 0.75$$
$$19.25 \leq x \leq 20.75$$

The weights can range between 19.25 ounces and 20.75 ounces, inclusive.

Example 5: Writing an Absolute Value Model

a. You are a quality control inspector at a bowling pin company. A regulation pin must weigh between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.

| Weight of pin – Average of extreme weights | >Tolerance

Weight of pin = w

Average of extreme weights = $\frac{50+58}{2} = 54$

Tolerance = 58-54 = 4

$|w - 54| > 4$

You should reject a bowling pin if its weight w satisfies $|w - 54| > 4$

Decide whether the number is a solution of the equation.

1. $|5x - 4| = 6$; 2

2. $|3x + 4| = 8$; -4

3. $|2x - 3| = 7$; 2

4. $|5 - 3x| = 8$; 1

5. $\left|\frac{1}{2}x - 2\right| = 4$; -1

6. $\left|3 - \frac{1}{4}x\right| = 4$; 28

Solve the equation.

7. $|x + 3| = 5$

8. $|3x - 2| = 8$

9. $|2x + 6| = 14$

10. $\left|\frac{1}{2}t - 4\right| = 1$

11. $|11 - 3t| = 2$

12. $|7t + 3| = 4$

13. $|2x - 7| = 7$

14. $\left|1 - \frac{2}{3}x\right| = 9$

15. $|4 - 5x| = 6$

Solve the inequality.

16. $|x - 5| < 1$

17. $|3x + 2| \le 7$

18. $|4 - x| < 5$

19. $|x + 8| \ge 3$

20. $|2x - 1| > 5$

21. $|11 - 3x| > 4$

22. $\left|\frac{1}{2}x - 3\right| \le 5$

23. $\left|2 - \frac{1}{3}x\right| \ge 10$

24. $|4x - 1| < 3$

25. **Touring a Ship** The diagram below shows the water line of a large ship. The ship extends 27 feet above the water and 27 feet below the water. Suppose you toured the entire ship. Write an absolute value inequality that represents all the distances you could have been from the water line.

```
                         27 ft

                         0 ft

                        -27 ft
```
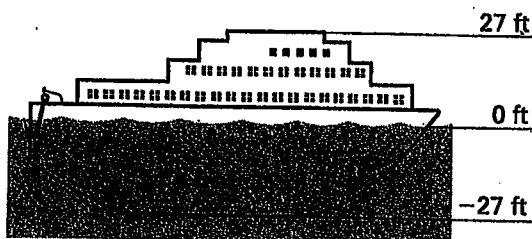
26. **Water Temperature** Most fish can adjust to a change in the water temperature of up to 15° F if the change is not sudden. Suppose a lake trout is living comfortably in water that is 58° F. Write an absolute value inequality that represents the range of temperatures at which the lake trout can survive.
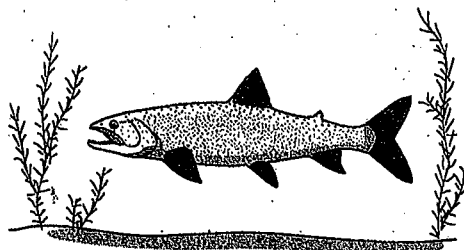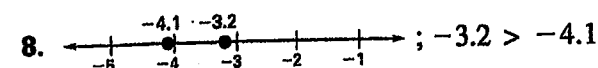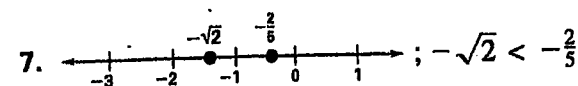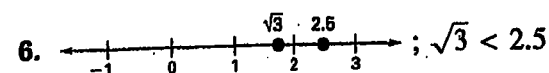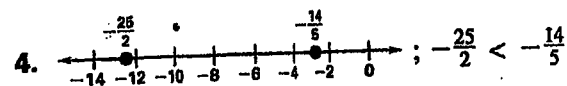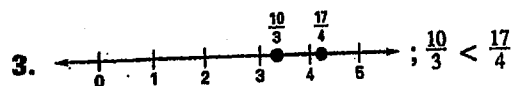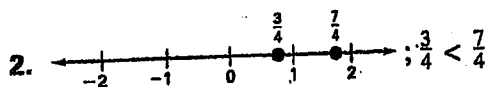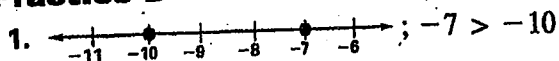
27. **Hours of Daylight** According to the *Old Farmer's Almanac*, the hours of daylight in Fairbanks, Alaska, range from approximately $3\frac{1}{2}$ hours in mid-December to approximately 21 hours in mid-June. Write an absolute value inequality that represents the hours of daylight in Fairbanks.

28. **Elephant Longevity** On average an elephant will live from 30 to 40 years. Write an absolute value inequality that represents the typical ages of an elephant.

Lesson 1.1

Practice B

1. $; -7 > -10$

2. $; \frac{3}{4} < \frac{7}{4}$

3. $; \frac{10}{3} < \frac{17}{4}$

4. $; -\frac{25}{2} < -\frac{14}{5}$

5. $; 0.8 > -0.9$

6. $; \sqrt{3} < 2.5$

7. $; -\sqrt{2} < -\frac{2}{5}$

8. $; -3.2 > -4.1$

9. $; -\frac{8}{3} < -\frac{7}{5}$
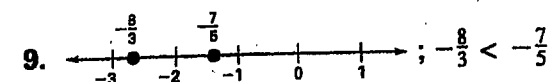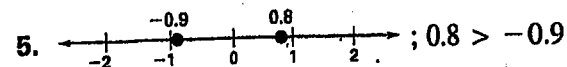
10. $-4, -\frac{3}{2}, \frac{1}{2}, 1, 2$ 11. $-\frac{1}{3}, 0, \frac{3}{4}, \frac{7}{2}$

12. $-3, -\sqrt{7}, \frac{1}{5}, 2.1$ 13. $-\sqrt{10}, -2.9, \frac{15}{2}, 8$

14. $-\frac{5}{2}, -\sqrt{3}, 1, \sqrt{2}, 3$

15. $-6, -\sqrt{5}, -\frac{1}{2}, \frac{7}{3}, \frac{13}{4}$

16. Identity property of multiplication

17. Commutative property of addition

18. Inverse property of addition

19. Associative property of multiplication

20. Associative property of addition

21. Identity property of addition

22. -5 23. -20 24. -4 25. -3 26. 24

27. 27 28. -4 29. 18 30. 52 in.

31. 3 touchdowns 32. 3 pieces

33. -2 or 2 under par

Lesson 1.2

Practice B

1. a^5 2. $(-9)^4$ 3. $(-x)^3$ 4. $(2y)^3 + 7$

5. $(4b)^2 + (2a)^2$ 6. 8^n 7. 81 8. -64

9. 32 10. 5 11. 8 12. 24 13. 15

14. 32 15. -3 16. 15 17. 7 18. 28

19. $-\frac{5}{2}$ 20. 9 21. 12 22. -2 23. -2

24. 2 25. 4 26. $2y(x + y)$; 30

27. $\frac{1}{2}x(4x + 1)$; 75 28. $8.95x + 29.95$; \$65.75

29. $6.95x + 24.99(5 - x)$; \$70.83

30. \$270; \$415

Lesson 1.3

Practice B

1. 20 2. 2 3. 3 4. 2 5. 4 6. $\frac{1}{4}$ 7. $\frac{14}{3}$

8. -5 9. $\frac{5}{2}$ 10. -3 11. -20 12. -24

13. $\frac{1}{7}$ 14. -13 15. $\frac{4}{7}$ 16. $\frac{4}{3}$ 17. 0

18. $-\frac{27}{2}$ 19. $\frac{29}{3}$ 20. 1 21. $\frac{7}{6}$

22. $2x - 3 = 11$ ft; $3x - 5 = 16$ ft;
    $15 - x = 8$ ft

23. $15 - 2x = 9$ ft; $x + 7 = 10$ ft

24. \$22 25. 3 tickets 26. 7.5 hours

27. 2.75 hours 28. 4.2 hours 29. 3 children

Lesson 1.4

Practice B

1. -2 **2.** 2 **3.** $\frac{13}{2}$ **4.** 8 **5.** 4

6. 1 **7.** $y = \frac{2}{3}x - 1;\ -3$

8. $y = -\frac{3}{2}x + 2;\ -4$ **9.** $y = -\frac{8}{3}x + \frac{10}{3};\ -\frac{22}{3}$

10. $y = \frac{5x - 2}{x};\ 7$ **11.** $y = -\frac{21}{16}x - \frac{21}{2};\ -21$

12. $y = \frac{27}{10}x + \frac{117}{2};\ 99$ **13.** $s = \frac{2}{\sqrt{3}}h$

14. $s = \frac{P}{3}$ **15.** $h = \frac{3V}{\pi r^2}$ **16.** $C = \frac{5}{9}(F - 32)$

17. $h = \frac{2A}{b_1 + b_2}$ **18.** $b_2 = \frac{2A}{h} - b_1$

19. $r = \frac{S}{2\pi h}$ **20.** $h = \frac{V}{\pi r^2}$ **21.** $s = \frac{P}{4};\ 11$ cm

22. $l = \frac{A}{w};\ 3$ ft **23.** $r^3 = \frac{3V}{4\pi};\ 4$ m

24. 8 m **25.** 64π m$^2 \approx 201.1$ m^2

Lesson 1.5

Practice B

1. Distance traveled $= 100$ miles
Rate of travel $= 763$ miles per hour
Time traveled $= t$

2. $100 = 763t$ **3.** About 0.131

4. About 0.131 hour or about 7.86 minutes

5. | Total cost | $=$ | Price per square yard | $\cdot$ | Number of square yards |

6. Total cost $= \$450$
Price per square yard $= x$
Number of square yards $= 30$ square yards

7. $450 = 30x$ **8.** 15 **9.** $\$15$ per square yard

10. | Distance traveled | $=$ |
| Your speed | $\cdot$ | Your time | $+$ | Friend's speed | $\cdot$ | Friend's time |

11. Distance traveled $= 300$ miles
Your speed $= r$
Your time $= 3$ hours
Friend's speed $= 52$ miles per hour
Friend's time $= 3$ hours
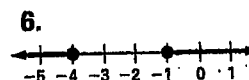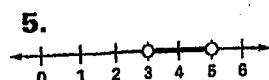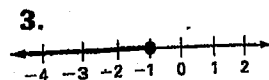
12. $300 = 3r + 156$ **13.** 48

14. 48 miles per hour

15.
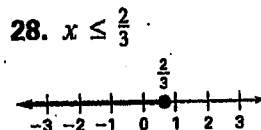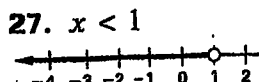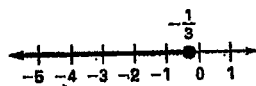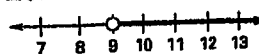| Total time | $=$ | Time per trial | $\cdot$ | Number of trials | $+$ | Time to write report |

16. Total time $= 1\frac{1}{2}$ hours or 90 minutes
Time per trial $= 5$ minutes
Number of trials $= x$
Time to write report $= 30$ minutes

17. $90 = 5x + 30$ **18.** 12 **19.** 12 trials

Lesson 1.6

Practice B

1.

2.

3.

4.

5.

6.
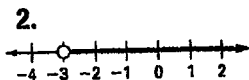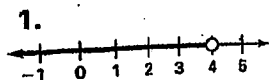
7. $x < 6$ **8.** $x < -7$ **9.** $x > -2$

10. $x < 2$ **11.** $x \geq 1$ **12.** $x \leq 6$ **13.** $x \leq -1$

14. $x \geq 2$ **15.** $x < 2$ **16.** $x > -\frac{5}{9}$

17. $x \leq 0$ **18.** $x \leq 3$

19. $\frac{3}{2} < x < 4$ **20.** $-4 < x < 6$

21. $x \leq 6$ or $x \geq 16$ **22.** $x < -2$ or $x > 4$

23. $8 \leq x \leq 12$ **24.** $x < 7$

25. $x > 9$

26. $x \leq -\frac{1}{3}$

27. $x < 1$

28. $x \leq \frac{2}{3}$

29. $x < -3$

30. $x \leq 1$
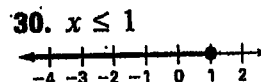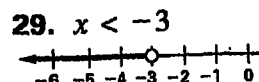
31. $18°55' \leq x \leq 71°23'$

32. $x + 444 \geq 540;\ x \geq 96$

33. $d \leq 65(2);\ d \leq 130$ **34.** $-80 \leq x \leq 98$

35. $0.4 \leq x \leq 8$

Lesson 1.7

Practice B

1. yes **2.** yes **3.** no **4.** no **5.** no **6.** yes

7. $-8, 2$ **8.** $-2, \frac{10}{3}$ **9.** $-10, 4$ **10.** $6, 10$

11. $3, \frac{13}{3}$ **12.** $-1, \frac{1}{7}$ **13.** $0, 7$ **14.** $-12, 15$

15. $-\frac{2}{5}, 2$ **16.** $4 < x < 6$ **17.** $-3 \leq x \leq \frac{5}{3}$

18. $-1 < x < 9$ **19.** $x \leq -11$ or $x \geq -5$

20. $x < -2$ or $x > 3$ **21.** $x < \frac{7}{3}$ or $x > 5$

22. $-4 \leq x \leq 16$ **23.** $x \leq -24$ or $x \geq 36$

24. $-\frac{1}{2} < x < 1$ **25.** $|x| \leq 27$

26. $|x - 58| \leq 15$ **27.** $|x - 12.25| \leq 8.75$

28. $|x - 35| \leq 5$