NESHAMINY HIGH SCHOOL

AP CALCULUS

SUMMER REVIEW PACKET

Congratulations on accepting the challenge of taking the Advanced Placement Calculus course at Neshaminy High School. AP Calculus is a rigorous and demanding college-level math course. Towards the end of the course students will be able to take the Calculus-AB AP test. Students who perform well on this standardized test may be eligible to receive college credit and/or bypass introductory calculus classes at college. The exact amount of credit will depend on the test score, and on which college they choose to attend.

The problems in this packet are designed to help you review topics from Algebra and Pre-Calculus which are important to your success in AP Calculus. All too often, we have seen students show perfect calculus work, only to get the final answer incorrect due to algebra or trigonometry errors. Work on this packet at your leisure during the summer to maintain the pre-requisite skills necessary for success in AP Calculus. This packet will be collected during the first week of classes as a homework assignment.

Have a wonderful and relaxing summer.

Neshaminy High School Mathematics Department

I. Complex Fractions:

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

II. Functions:

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$
\begin{aligned}
f(g(x)) &= f(x-4) \\
&= 2(x-4)^2 + 1 \\
&= 2(x^2 - 8x + 16) + 1 \\
&= 2x^2 - 16x + 32 + 1 \\
f(g(x)) &= 2x^2 - 16x + 33
\end{aligned}
$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

1. $f(2)$
2. $g(-3)$
3. $f(t+1)$
4. $f(g(-2))$
5. $g(f(m+2))$
6. $\dfrac{f(x+h) - f(x)}{h}$

Let $f(x) = \sin x$. Find each.

7. $f\left(\dfrac{\pi}{2}\right)$
8. $f\left(\dfrac{2\pi}{3}\right)$

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

9. $h(f(-2))$
10. $f(g(x-1))$
11. $g(h(x^3))$

Find $\dfrac{f(x+h) - f(x)}{h}$ for the given function.

12. $f(x) = 9x + 3$
13. $f(x) = 5 - 2x$

III. Intercepts:

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$\underline{x - \text{int}. \ (Let \ y = 0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1 \ or \ x = 3$

$x - \text{intercepts} \ (-1,0) \ and \ (3,0)$

$\underline{y - \text{int}. \ (Let \ x = 0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept} \ (0,-3)$

Find the x and y intercepts for each.

1. $y = 2x - 5$

2. $y = x^2 + x - 2$

3. $y = x\sqrt{16 - x^2}$

4. $y^2 = x \ \ - 4x$

IV. Points of Intersection:

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2+y^2-16x+39=0$$
$$x^2-y^2-9=0$$

<u>Elimination Method</u>

$2x^2-16x+30=0$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$x=3$ *and* $x=5$

Plug x=3 *and* $x=5$ into one original

$3^2-y^2-9=0$ $\quad$ $5^2-y^2-9=0$

$-y^2=0$ $\quad$ $16=y^2$

$y=0$ $\quad$ $y=\pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

<u>Substitution Method</u>

Solve one equation for one variable.

$y^2=-x^2+16x-39$ (1st equation solved for y)

$x^2-(-x^2+16x-39)-9=0$ Plug what y^2 is equal to into second equation.

$2x^2-16x+30=0$ (*The rest is the same as previous example*)

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$x=3$ *or* $x-5$

Find the point(s) of intersection of the graphs for the given equations.

1. $$\begin{aligned} x+y&=8 \\ 4x-y&=7 \end{aligned}$$

2. $$\begin{aligned} x^2+y&=6 \\ x+y&=4 \end{aligned}$$

3. $$\begin{aligned} x^2-4y-20x-64y-172&=0 \\ 16x^2+4y^2-320x+64y+1600&=0 \end{aligned}$$

V. Interval Notation:

1. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1, 7)$	
		

Solve each inequality. State your answer in BOTH interval notation and graphically.

2. $2x - 1 \ge 0$

3. $-4 \le 2x - 3 < 4$

4. $\dfrac{x}{2} - \dfrac{x}{3} > 5$

VI. Domain & Range:

Find the domain and range of each function. Write your answer in INTERVAL notation.

1. $f(x) = x^2 - 5$

2. $f(x) = -\sqrt{x+3}$

3. $f(x) = 3\sin x$

4. $f(x) = \dfrac{2}{x-1}$

VII. Inverses:

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse of each function.

1. $f(x)=2x+1$

2. $f(x)=\dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x))=g(f(x))=x$

Example:

If: $f(x)=\dfrac{x-9}{4}$ *and* $g(x)=4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x))=4\left(\frac{x-9}{4}\right)+9$$
$$=x-9+9$$
$$=x$$

$$g(f(x))=\frac{(4x+9)-9}{4}$$
$$=\frac{4x+9-9}{4}$$
$$=\frac{4x}{4}$$
$$=x$$

$f(g(x))=g(f(x))=x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

3. $f(x)=\dfrac{x^3}{2}$ $\quad g(x)=\sqrt[3]{2x}$

4. $f(x)=9-x^2\ ,\ x\ge 0$ $\quad g(x)=\sqrt{9-x}$

VIII. Equations of Lines:

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

1. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

2. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

3. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

4. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of $\frac{2}{3}$.

5. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

6. Find the equation of a line perpendicular to the y-axis passing through the point (4, 7).

7. Find the equation of a line passing through the points (-3, 6) and (1, 2).

8. Determine the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

IX. Radian & Degree Measure:

Use $\frac{\text{degrees}}{180} = \frac{\text{radians}}{\pi}$ to convert back and forth between degrees and radians. OR
Multiply by $\frac{180^\circ}{\pi}$ to convert to degrees and multiply by $\frac{\pi}{180^\circ}$ to convert to radians.

Convert to degrees.

1. $\frac{5\pi}{6}$

2. $\frac{4\pi}{5}$

Convert to radians.

3. 45°

4. 237°

X. Angles In Standard Position:

Sketch the angle in standard position.

1. $\frac{11\pi}{6}$

2. 230°

3. $-\frac{5\pi}{3}$

XI. Reference Triangles:

Sketch the angle in standard position. Draw the reference triangle and label the sides.

1. $\frac{2\pi}{3}$

2. 225°

3. $-\frac{\pi}{4}$

4. 30°

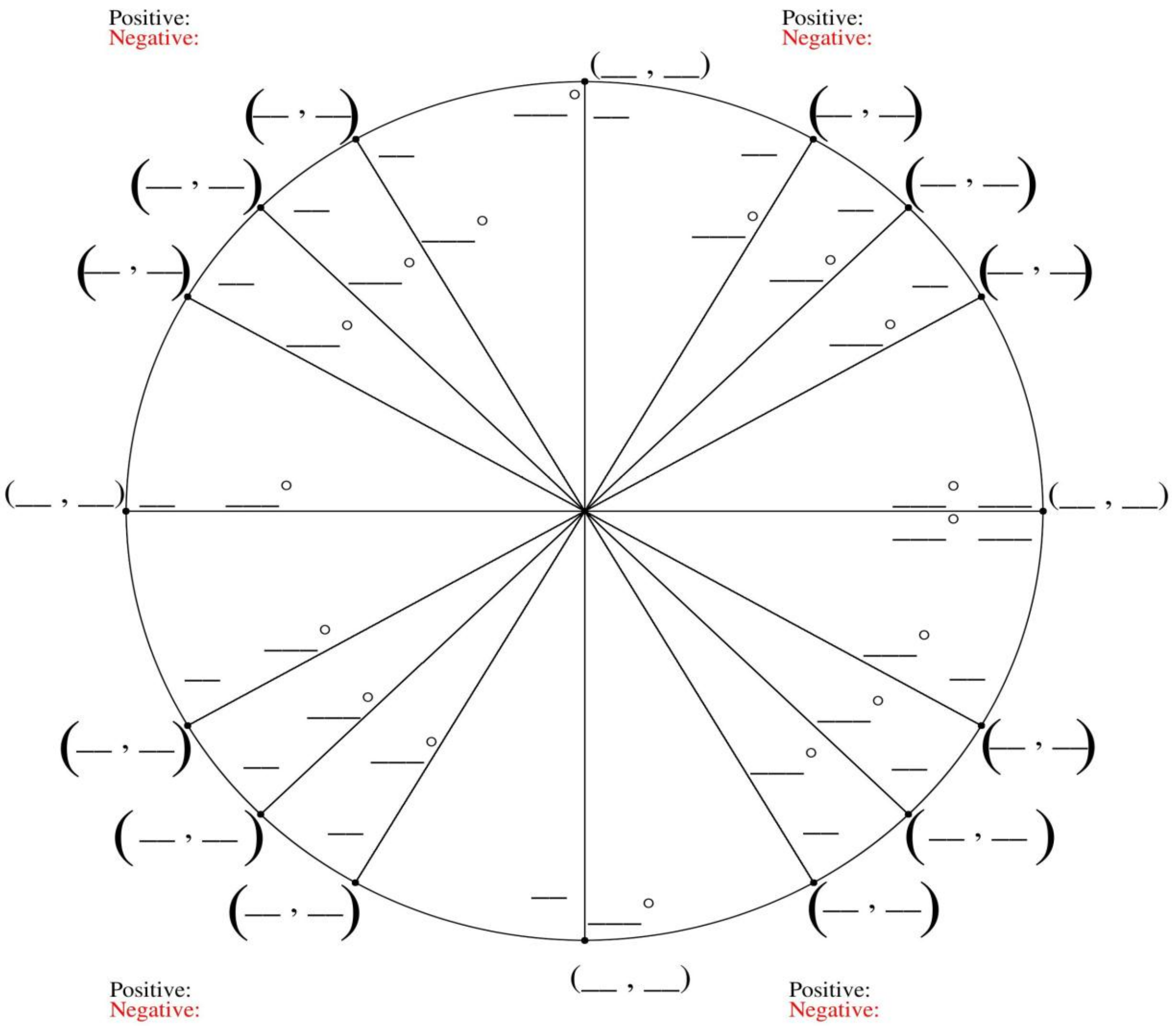
Circle with Radius 1

otherwise known as unit circle

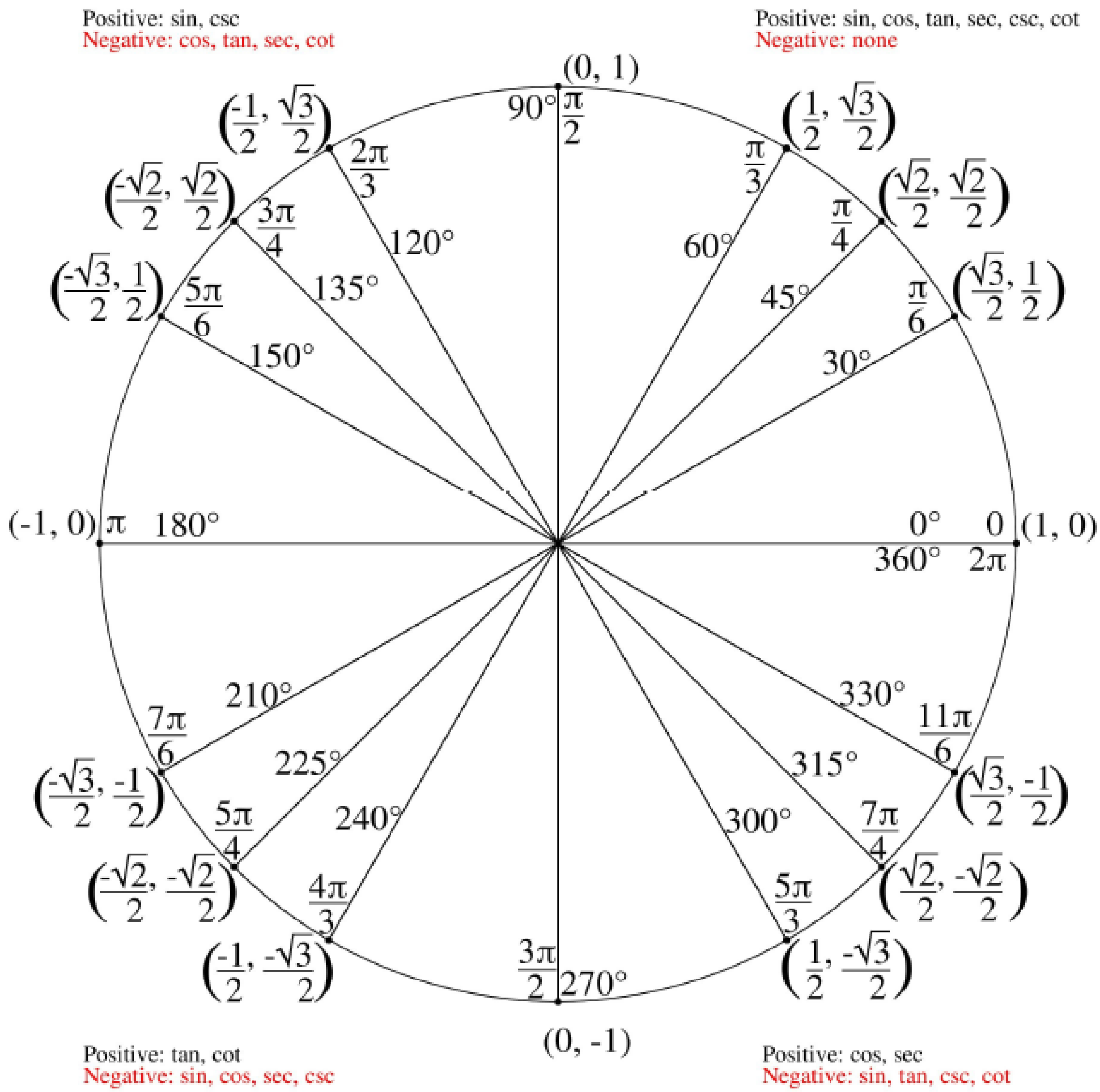
$$(x,y) = (\cos\theta, \sin\theta)$$

You can determine the sine or cosine of an angle by using the unit circle. The x coordinate of the circle is cosine, and the y-coordinate is the sine.

Fill in the (x, y) coordinates below for the Unit Circle. The answer key is on the next page.



Answer Key for Unit Circle:



XII. Trigonometric Equations:

Solve each equation for x. ($0 \leq x < 2\pi$)

1. $\sin x = -\frac{1}{2}$

2. $2\cos x = \sqrt{3}$

3. $\cos 2x = \frac{1}{\sqrt{2}}$

4. $\sin^2 x = \frac{1}{2}$

5. $\sin 2x = -\frac{\sqrt{3}}{2}$

6. $2\cos^2 x - 1 - \cos x = 0$

7. $4\cos^2 x - 3 = 0$

8. $\sin^2 x + \cos 2x - \cos x = 0$

XIII. Simplifying Expressions:

Examples:

$\sqrt[3]{(2)^3} = 2$

$\sqrt[3]{8} = 8^{1/3} = 2$

$\left(\sqrt[10]{25}\right)^5 = \left(25^{1/10}\right)^5 = 25^{5/10} = 25^{1/2} = \sqrt{25} = 5$

$x^{-4} = \dfrac{1}{x^4}$

$\dfrac{1}{x^{-3}} = x^3$

$(3x)^{-2} = \dfrac{1}{(3x)^2} = \dfrac{1}{9x^2}$

$x^6 x^5 = x^{11}$

$\dfrac{x^3}{x^9} = x^{-6} = \dfrac{1}{x^6}$

$\left(x^2\right)^3 = x^6$

Simplify each expression. Write answers with positive exponents where applicable.

1. $\dfrac{1}{x+h} - \dfrac{1}{x}$

2. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^3}}$

3. $\dfrac{12x^{-3}y^2}{18xy^{-1}}$

4. $\dfrac{15x^2}{5\sqrt{x}}$

5. $(5a^3)(4a^2)$

6. $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

7. $\dfrac{\frac{1}{2} - \frac{5}{4}}{\frac{3}{8}}$

8. $\dfrac{5-x}{x^2-25}$

9. $\sqrt[3]{128}$

XIV. Logarithms:

There are **TWO** bases that are used for logarithms

Common Log **(Base 10)**

$y = \log x$ $\quad (\text{same as } 10^y = x)$

Natural Log **(Base e)**

$y = \ln x$ $\quad (\text{same as } e^y = x)$

$y = \log_a x$ ← argument

answer ↑ $\quad$ ↑ base

Exponential Form **Logarithmic Form**

$y = a^x \longrightarrow$ $\boxed{\text{Inverse}}$ $\longleftarrow y = \log_a x$

<u>**Properties of Logarithmic Expansion**</u>		<u>**Exponential Equivalent**</u>
1.	$\log_a 1 = 0$ and $\ln 1 = 0$	- comes from the facts that $a^0 = 1$ and $e^0 = 1$
2.	$\log_a a = 1$ and $\ln e = 1$	- comes from the facts that $a^1 = a$ and $e^1 = e$
3.	$a^{\log_a M} = M$ and $e^{\ln M} = M$ $\log_a M = \log_a M$ and $\ln_e M = \ln M$	- comes from rewriting each as a logarithmic function
4.	$\log_a a^r = r$ and $\ln e^r = r$ $a^r = a^r$ and $e^r = e^r$ (remember the understood e in the ln)	- comes from rewriting logarithm in exponential form
5.	$\log_a (MN) = \log_a M + \log_a N$	- comes from the exponent rule $a^M \cdot a^N = a^{M+N}$
6.	$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$	- comes from the exponent rule $\frac{a^M}{a^N} = a^{M-N}$
7.	$\log_a M^r = r \log_a M$	- proved by using $x = \log_a M$ rewritten exponentially, each side raised to the r power then simplified
8.	if $\log_a M = \log_a N$ then M=N	- comes from the exponent rule if $a^M = a^N$ then M=N

Solve.

1. $\log_a x + \log_a (x-2) = \log_a (x+4)$

2. $2\log_3 (x+4) - \log_3 9 = 2$

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x\cos x$ $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

$y = \ln x = \log_e x$ is equivalent to $x = e^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p\log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h\to\infty} \dfrac{f(x+h)-f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0