Warm Up
A figure has vertices A, B, and C. After a transformation, the image of the figure has vertices A′, B′, and C′. Draw the pre-image and the image on graph paper. Then identify the transformation.

1. A(-3, 1), B(-1, 1), C(-3, 4) translation 6 units right
   A′(3, 1), B′(5, 1), C′(3, 4)

2. A(2, 1), B(5, 1), C(4, 3) reflection across x-axis
   A′(2, -1), B′(5, -1), C′(4, -3)
4-1 Congruence and Transformations

**Objectives**

Draw, identify, and describe transformations in the coordinate plane.

Use properties of rigid motions to determine whether figures are congruent and to prove figures congruent.
Vocabulary

dilation
isometry
rigid transformation
A **dilation** with scale factor $k > 0$ and center $(0, 0)$ maps $(x, y)$ to $(kx, ky)$. 
Remember!

In a transformation, the original figure is the pre-image. The resulting figure is the image.
Example 1: Drawing and Identifying Transformations

Apply the transformation \( M \) to the polygon with the given vertices. Identify and describe the transformation.

A. \( M: (x, y) \rightarrow (x - 4, y + 1) \)

\( P(1, 3), Q(1, 1), R(4, 1) \)

translation 4 units left and 1 unit up
Example 1: Continued

B. \( M: (x, y) \rightarrow (x, -y) \)

\( A(1, 2), B(4, 2), C(3, 1) \)

reflection across x-axis
Example 1: Continued

C. $M: (x, y) \rightarrow (y, -x)$

$R(-3, 0), E(-3, 3), C(-1, 3), T(-1, 0)$

$90^\circ$ rotation clockwise with center of rotation $(0, 0)$
Example 1: Continued

D. \( M: (x, y) \rightarrow (3x, 3y) \)

\( K(-2, -1), L(1, -1), N(1, -2) \) (dilation with scale factor 3 and center (0, 0))
Check It Out! Example 1

1. Apply the transformation \( M : (x, y) \rightarrow (3x, 3y) \) to the polygon with vertices \( D(1, 3), E(1, -2), \) and \( F(3, 0) \). Name the coordinates of the image points. Identify and describe the transformation.

\[ D'(3, 9), E'(3, -6), F'(9, 0); \text{ dilation with scale factor 3} \]
# Representing Transformations in the Coordinate Plane

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<th>TRANSFORMATION</th>
<th>COORDINATE MAPPING AND DESCRIPTION</th>
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<tr>
<td>Translation</td>
<td>((x, y) \rightarrow (x + a, y + b)) Translation (a) units horizontally and (b) units vertically</td>
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<tr>
<td>Reflection</td>
<td>((x, y) \rightarrow (-x, y)) Reflection across y-axis</td>
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<td></td>
<td>((x, y) \rightarrow (x, -y)) Reflection across x-axis</td>
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<td>((x, y) \rightarrow (y, -x)) Rotation about (0, 0), 90° clockwise</td>
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<td>Dilation</td>
<td>((x, y) \rightarrow (kx, ky), k &gt; 0) Dilation with scale factor (k) and center (0, 0)</td>
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An **isometry** is a transformation that preserves length, angle measure, and area. Because of these properties, an isometry produces an image that is congruent to the preimage.

A **rigid transformation** is another name for an isometry.
Transformations and Congruence

Translations, reflections, and rotations produce images that are congruent to their preimages.

Dilations with scale factor $k \neq 1$ produce images that are not congruent to their preimages.
Example 2: Determining Whether Figures are Congruent

Determine whether the polygons with the given vertices are congruent.

A. \( A(-3, 1), B(2, 3), C(1, 1) \)

\( P(-4, -2), Q(1, 0), R(0, -2) \)

The triangle are congruent; \( \triangle ABC \) can be mapped to \( \triangle PQR \) by a translation: \((x, y) \rightarrow (x - 1, y - 3)\).
The triangles are not congruent; \( \triangle ABC \) can be mapped to \( \triangle PQR \) by a dilation with scale factor \( k \neq 1: (x, y) \rightarrow (1.5x, 1.5y) \).
Determine whether the polygons with the given vertices are congruent. Support your answer by describing a transformation: A(2, -1), B(3, 0), C(2, 3) and P(1, 2), Q(0, 3), R(-3, 2).

The triangles are congruent because \( \triangle ABC \) can be mapped to \( \triangle PQR \) by a rotation: \((x, y) \rightarrow (-y, x)\).
Example 3: Applying Transformations

Prove that the polygons with the given vertices are congruent.

\[ A(1, 2), B(2, 1), C(4, 2) \]
\[ P(-3, -2), Q(-2, -1), R(-3, 1) \]

\( \triangle ABC \) can be mapped to \( \triangle A'B'C' \) by a translation: \((x, y) \rightarrow (x - 3, y + 1)\); and then \( \triangle A'B'C' \) can be mapped to \( \triangle PQR \) by a rotation: \((x, y) \rightarrow (-y, x)\).
Check It Out! Example 3

Prove that the polygons with the given vertices are congruent: \( A(-4, -2), B(-2, 1), C(2, -2) \) and \( P(1, 0), Q(3, -3), R(3, 0) \).

The triangles are congruent because \( \triangle ABC \) can be mapped to \( \triangle A'B'C' \) by a translation \((x, y) \rightarrow (x + 5, y + 2)\); and then \( \triangle A'B'C' \) can be mapped to \( \triangle ABC \) by a reflection across the x-axis.
Translations, reflections, and rotations can be called congruence transformations.
Example 4: Architecture Application

Is there another transformation that can be used to create this frieze pattern? Explain your answer.
Repeated reflections can create this frieze pattern; a reflection of any section over a line through either the left or right side of each section.
Sketch a frieze pattern that can be produced by using reflections

Possible answer: repeated horizontal reflections
Lesson Quiz : Part-I

Apply the transformation $M$ to the polygon with the given vertices. Identify and describe the transformation.

1. $M: (x, y) \rightarrow (3x, 3y)$
   
   $A(0, 1), B(2, 1), C(2, -1)$
   
   dilation with scale factor 3 and center (0, 0)

2. $M: (x, y) \rightarrow (-y, x)$
   
   $A(0, 3), B(1, 2), C(4, 5)$
   
   $90^\circ$ rotation counterclockwise with center of rotation (0, 0)
3. \( M: (x, y) \rightarrow (x + 1, y - 2) \)
   
   \( A(-2, 1), B(-2, 4), C(0, 3) \)

   translation 1 unit right and 2 units down

4. Determine whether the triangles are congruent.
   \( A(1, 1), B(1, -2), C(3, 0) \)
   \( J(2, 2), K(2, -4), L(6, 0) \)

   not \( \cong \); \( \triangle ABC \) can be mapped to \( \triangle JKL \) by a dilation with scale factor \( k \neq 1: (x, y) \rightarrow (2x, 2y) \).
5. Prove that the triangles are congruent. A(1, -2), B(4, -2), C(1, -4) D(-2, 2), E(-5, 2), F(-2, 0)

\[ \triangle ABC \text{ can be mapped to } \triangle A'B'C' \text{ by a translation: } (x, y) \rightarrow (x + 1, y + 4); \text{ and then} \]
\[ \triangle A'B'C' \text{ can be mapped to } \triangle DEF \text{ by a reflection: } (x, y) \rightarrow (-x, y). \]