3-3 Proving Lines Parallel

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

State the converse of each statement.

1. If $a = b$, then $a + c = b + c$.

If $a + c = b + c$, then $a = b$.

2. If $m\angle A + m\angle B = 90°$, then $\angle A$ and $\angle B$ are complementary.

If $\angle A$ and $\angle B$ are complementary, then $m\angle A + m\angle B = 90°$.

3. If $AB + BC = AC$, then A, B, and C are collinear.

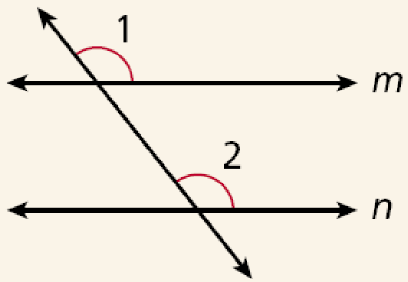
If A, B, and C are collinear, then $AB + BC = AC$.

Objective

Use the angles formed by a transversal to prove two lines are parallel.

Recall that the converse of a theorem is found by exchanging the hypothesis and conclusion. The converse of a theorem is not automatically true. If it is true, it must be stated as a postulate or proved as a separate theorem.

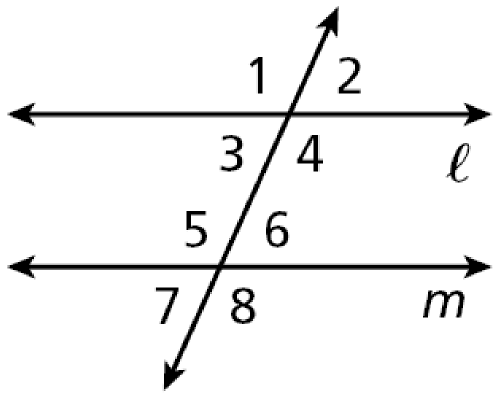
Postulate 3-3-1 Converse of the Corresponding Angles Postulate

THEOREM	HYPOTHESIS	CONCLUSION
If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$

Example 1A: Using the Converse of the Corresponding Angles Postulate

Use the Converse of the Corresponding Angles Postulate and the given information to show that ℓ || *m*.

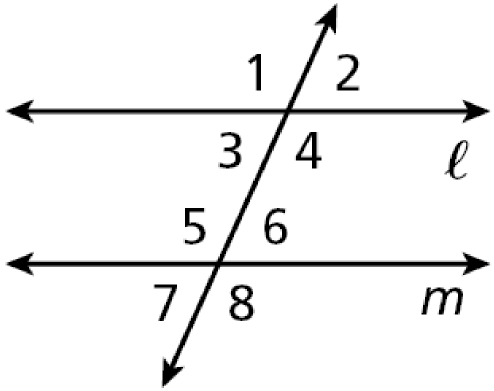
$\angle 4 \cong \angle 8$



$\angle 4 \cong \angle 8$	*∠4 and ∠8 are corresponding angles.*
ℓ || *m*	*Conv. of Corr. ∠s Post.*

Example 1B: Using the Converse of the Corresponding Angles Postulate

Use the Converse of the Corresponding Angles Postulate and the given information to show that ℓ || *m*.



$m\angle 3 = (4x - 80)°$,
$m\angle 7 = (3x - 50)°$, $x = 30$

$m\angle 3 = 4(30) - 80 = 40$ — *Substitute 30 for x.*

$m\angle 8 = 3(30) - 50 = 40$ — *Substitute 30 for x.*

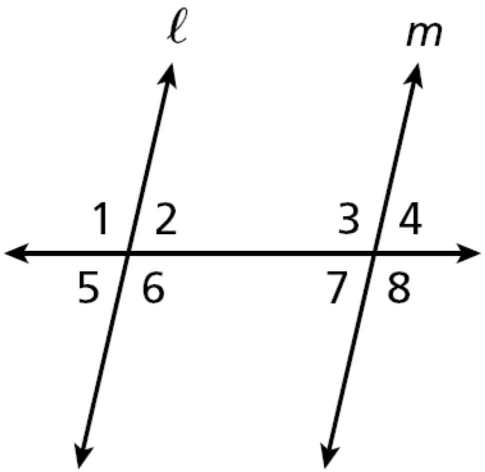
$m\angle 3 = m\angle 8$ — *Trans. Prop. of Equality*

$\angle 3 \cong \angle 8$ — *Def. of ≅ ∠s.*

ℓ || m — *Conv. of Corr. ∠s Post.*

Check It Out! Example 1a

Use the Converse of the Corresponding Angles Postulate and the given information to show that ℓ || *m*.



m∠1 = m∠3

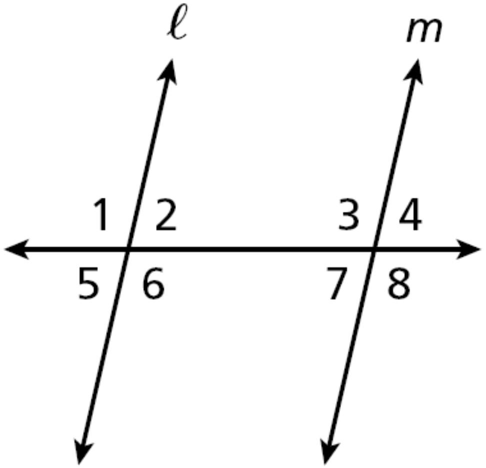
∠1 ≅ ∠3	*∠1 and ∠3 are corresponding angles.*
ℓ || *m*	*Conv. of Corr. ∠s Post.*

Check It Out! Example 1b

Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.

$m\angle 7 = (4x + 25)°$,

$m\angle 5 = (5x + 12)°$, $x = 13$



$m\angle 7 = 4(13) + 25 = 77$ — *Substitute 13 for x.*

$m\angle 5 = 5(13) + 12 = 77$ — *Substitute 13 for x.*

$m\angle 7 = m\angle 5$ — *Trans. Prop. of Equality*

$\angle 7 \cong \angle 5$ — *Def. of $\cong$ $\angle$s.*

$\ell \parallel m$ — *Conv. of Corr. $\angle$s Post.*

Postulate 3-3-2 **Parallel Postulate**

Through a point *P* not on line ℓ, there is exactly one line parallel to ℓ.

The Converse of the Corresponding Angles Postulate is used to construct parallel lines. The Parallel Postulate guarantees that for any line ℓ, you can always construct a parallel line through a point that is not on ℓ.

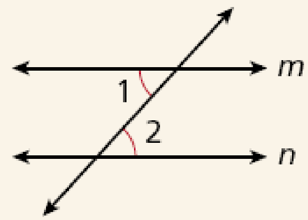
Theorems: Proving Lines Parallel

<table>
<tr><th>THEOREM</th><th>HYPOTHESIS</th><th>CONCLUSION</th></tr>
<tr><td>

3-3-3 Converse of the Alternate Interior Angles Theorem
If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.

</td><td>

$\angle 1 \cong \angle 2$



</td><td>$m \parallel n$</td></tr>
<tr><td>

3-3-4 Converse of the Alternate Exterior Angles Theorem
If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.

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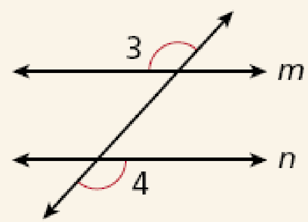
$\angle 3 \cong \angle 4$



</td><td>$m \parallel n$</td></tr>
<tr><td>

3-3-5 Converse of the Same-Side Interior Angles Theorem
If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.

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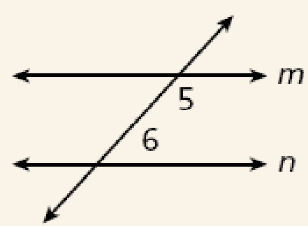
$m\angle 5 + m\angle 6 = 180°$

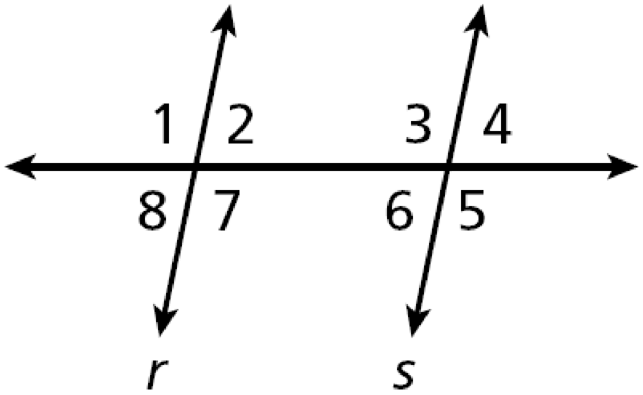


</td><td>$m \parallel n$</td></tr>
</table>

Example 2A: Determining Whether Lines are Parallel

Use the given information and the theorems you have learned to show that *r* || *s*.

∠4 ≅ ∠8



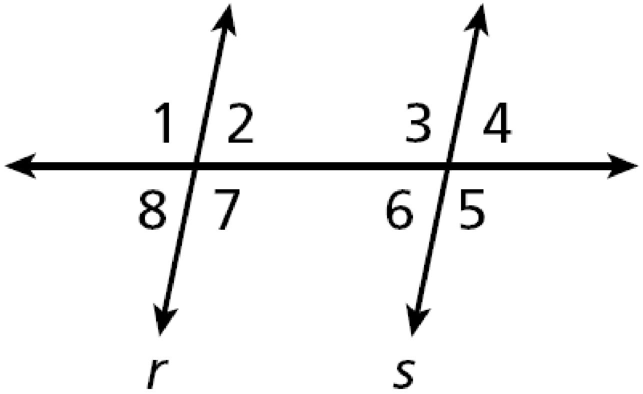
∠4 ≅ ∠8 — *∠4 and ∠8 are alternate exterior angles.*

r || *s* — *Conv. Of Alt. Int. ∠s Thm.*

Example 2B: Determining Whether Lines are Parallel

Use the given information and the theorems you have learned to show that *r* || *s*.

$m\angle 2 = (10x + 8)°$,
$m\angle 3 = (25x - 3)°$, $x = 5$



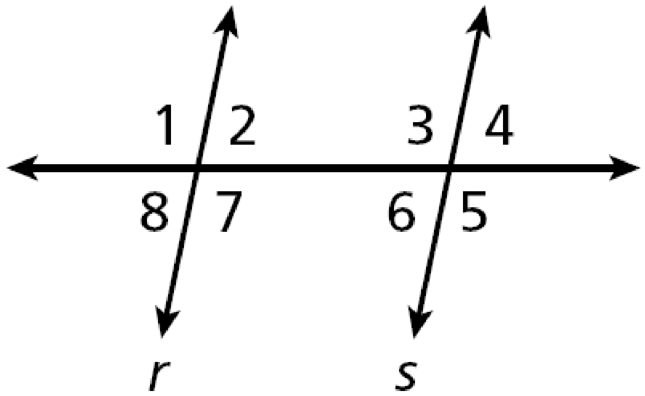
$m\angle 2 = 10x + 8$
$= 10(5) + 8 = 58$ *Substitute 5 for x.*

$m\angle 3 = 25x - 3$
$= 25(5) - 3 = 122$ *Substitute 5 for x.*

Example 2B Continued

Use the given information and the theorems you have learned to show that *r* || *s*.

$m\angle 2 = (10x + 8)°$,
$m\angle 3 = (25x - 3)°$, $x = 5$



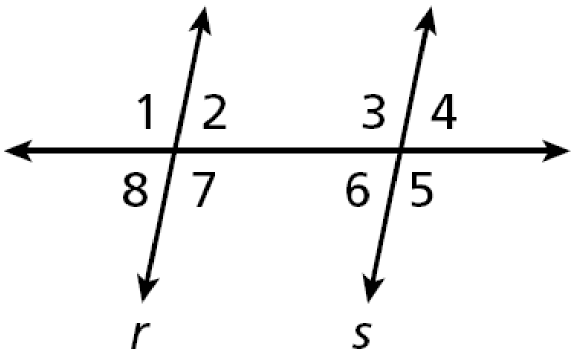
$m\angle 2 + m\angle 3 = 58° + 122°$
$= 180°$

∠2 and ∠3 are same-side interior angles.

r || *s* — *Conv. of Same-Side Int. ∠s Thm.*

Check It Out! Example 2a

Refer to the diagram. Use the given information and the theorems you have learned to show that *r* || *s*.



m∠4 = m∠8

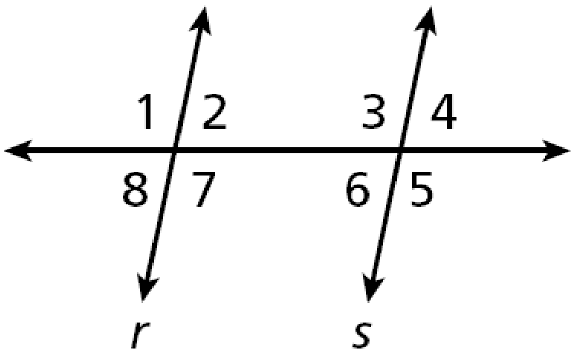
∠4 ≅ ∠8 *Congruent angles*

∠4 ≅ ∠8 *∠4 and ∠8 are alternate exterior angles.*

r || *s* *Conv. of Alt. Int. ∠s Thm.*

Check It Out! Example 2b

Refer to the diagram. Use the given information and the theorems you have learned to show that *r* || *s*.



m∠3 = 2*x*°, m∠7 = (*x* + 50)°, *x* = 50

m∠3 = 2*x*

= 2(50) = 100° *Substitute 50 for x.*

m∠7 = *x* + 50

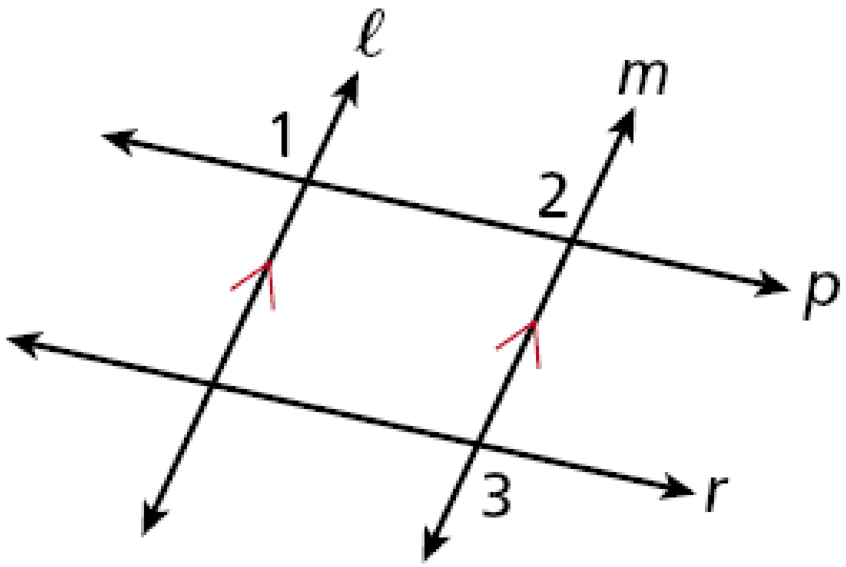
= 50 + 50 = 100° *Substitute 5 for x.*

m∠3 = 100° and m∠7 = 100°

∠3 ≅ ∠7 r||s *Conv. of the Alt. Int. ∠s Thm.*

Example 3: Proving Lines Parallel

Given: $p \parallel r$, $\angle 1 \cong \angle 3$
Prove: $\ell \parallel m$



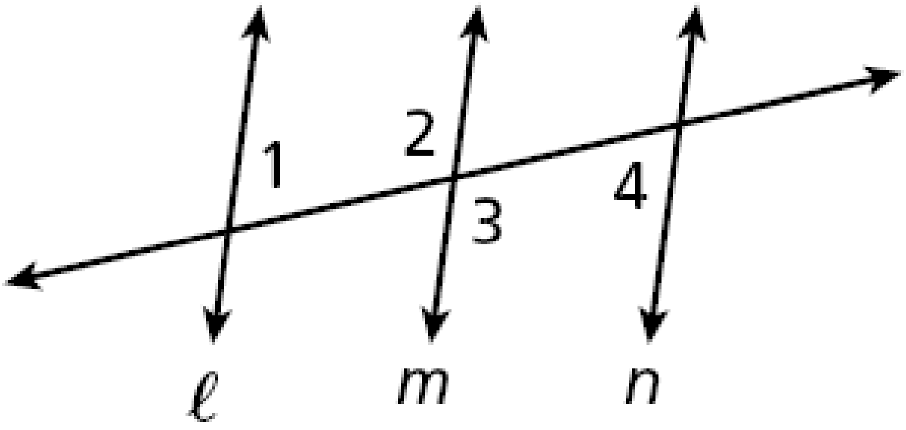
Example 3 Continued

Statements	Reasons
1. $p \parallel r$	**1.** Given
2. $\angle 3 \cong \angle 2$	**2.** Alt. Ext. ∠s Thm.
3. $\angle 1 \cong \angle 3$	**3.** Given
4. $\angle 1 \cong \angle 2$	**4.** Trans. Prop. of ≅
5. $\ell \parallel m$	**5.** Conv. of Corr. ∠s Post.

Check It Out! Example 3

Given: $\angle 1 \cong \angle 4$, $\angle 3$ and $\angle 4$ are supplementary.

Prove: $\ell \parallel m$

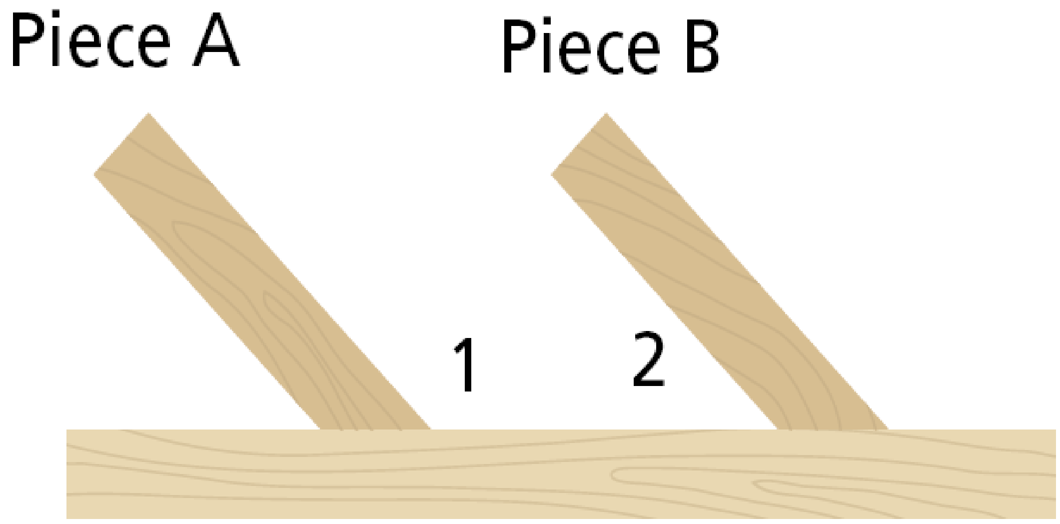


Check It Out! Example 3 Continued

Statements	Reasons
1. ∠1 ≅ ∠4	**1.** Given
2. m∠1 = m∠4	**2.** Def. ≅ ∠s
3. ∠3 and ∠4 are supp.	**3.** Given
4. m∠3 + m∠4 = 180°	**4.** Trans. Prop. of ≅
5. m∠3 + m∠1 = 180°	**5.** Substitution
6. m∠2 = m∠3	**6.** Vert.∠s Thm.
7. m∠2 + m∠1 = 180°	**7.** Substitution
8. ℓ || *m*	**8.** Conv. of Same-Side Interior ∠s Post.

Example 4: Carpentry Application

A carpenter is creating a woodwork pattern and wants two long pieces to be parallel. m∠1= $(8x + 20)°$ and m∠2 = $(2x + 10)°$. If $x = 15$, show that pieces A and B are parallel.



Example 4 Continued

A line through the center of the horizontal piece forms a transversal to pieces A and B.

$\angle 1$ and $\angle 2$ are same-side interior angles. If $\angle 1$ and $\angle 2$ are supplementary, then pieces A and B are parallel.

Substitute 15 for *x* in each expression.

Example 4 Continued

$m\angle 1 = 8x + 20$

$= 8(15) + 20 = 140$ *Substitute 15 for x.*

$m\angle 2 = 2x + 10$

$= 2(15) + 10 = 40$ *Substitute 15 for x.*

$m\angle 1 + m\angle 2 = 140 + 40$

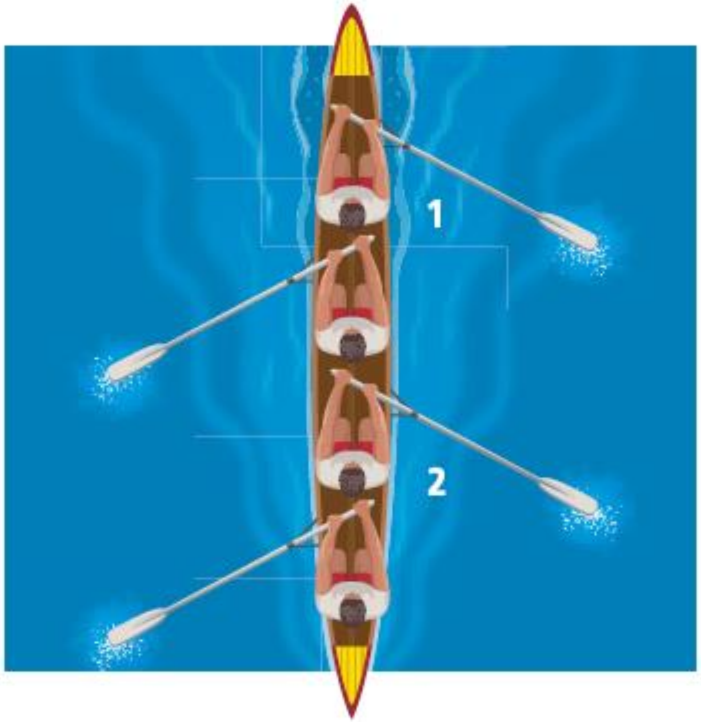
$= 180$

$\angle 1$ and $\angle 2$ are supplementary.

The same-side interior angles are supplementary, so pieces A and B are parallel by the Converse of the Same-Side Interior Angles Theorem.

Check It Out! Example 4

What if…? Suppose the corresponding angles on the opposite side of the boat measure $(4y - 2)°$ and $(3y + 6)°$, where $y = 8$. Show that the oars are parallel.

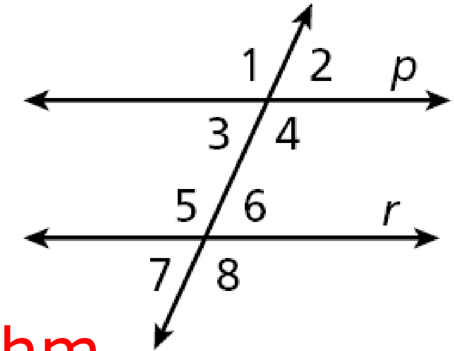


$4y - 2 = 4(8) - 2 = 30°$ $3y + 6 = 3(8) + 6 = 30°$

The angles are congruent, so the oars are || by the Conv. of the Corr. ∠s Post.

Lesson Quiz: Part I

Name the postulate or theorem that proves *p* || *r*.



1. ∠4 ≅ ∠5 Conv. of Alt. Int. ∠s Thm.

2. ∠2 ≅ ∠7 Conv. of Alt. Ext. ∠s Thm.

3. ∠3 ≅ ∠7 Conv. of Corr. ∠s Post.

4. ∠3 and ∠5 are supplementary.

Conv. of Same-Side Int. ∠s Thm.

Lesson Quiz: Part II

Use the theorems and given information to prove *p* || *r*.

5. m∠2 = (5*x* + 20)°, m ∠7 = (7*x* + 8)°, and *x* = 6

m∠2 = 5(6) + 20 = 50°
m∠7 = 7(6) + 8 = 50°
m∠2 = m∠7, so ∠2 ≅ ∠7

p || *r* by the Conv. of Alt. Ext. ∠s Thm.