In Exercises 1–4, find the derivative of the function by using the
definition of the derivative.

1. \( f(x) = x^2 - 4x + 5 \)
2. \( f(x) = \sqrt{x} + 1 \)
3. \( f(x) = \frac{x + 1}{x - 1} \)
4. \( f(x) = \frac{6}{x} \)

In Exercises 5 and 6, describe the \( x \)-values at which \( f \) is
differentiable.

5. \( f(x) = (x - 3)^{2/5} \)
6. \( f(x) = \frac{3x}{x + 1} \)

7. Sketch the graph of \( f(x) = 4 - |x - 2| \).
   (a) Is \( f \) continuous at \( x = 2 \)?
   (b) Is \( f \) differentiable at \( x = 2 \)? Explain.

8. Sketch the graph of \( f(x) = \frac{x^2 + 4x + 2}{1 - 4x - x^2}, \quad x < -2 \).
   (a) Is \( f \) continuous at \( x = -2 \)?
   (b) Is \( f \) differentiable at \( x = -2 \)? Explain.

In Exercises 9 and 10, find the slope of the tangent line to the
graph of the function at the given point.

9. \( g(x) = \frac{2}{3}x^2 - \frac{x}{6}, \quad (-1, \frac{5}{6}) \)
10. \( h(x) = \frac{3x}{8} - 2x^2, \quad (-2, \frac{35}{4}) \)

In Exercises 11 and 12, (a) find an equation of the tangent line
to the graph of \( f \) at the given point, (b) use a graphing utility to
graph the function and its tangent line at the point, and (c) use the
derivative feature of the graphing utility to confirm your results.

11. \( f(x) = x^3 - 1, \quad (1, -2) \)
12. \( f(x) = \frac{2}{x + 1}, \quad (0, 2) \)

In Exercises 13 and 14, use the alternative form of the derivative
to find the derivative at \( x = c \) (if it exists).

13. \( g(x) = x^2(x - 1), \quad c = 2 \)
14. \( f(x) = \frac{1}{x + 4}, \quad c = 3 \)

In Exercises 15–30, use the rules of differentiation to find the
dervative of the function.

15. \( y = 25 \)
16. \( y = -30 \)
17. \( f(x) = x^8 \)
18. \( g(x) = x^{20} \)
19. \( h(t) = 13t^4 \)
20. \( f(t) = -8t^2 \)
21. \( f(x) = x^3 - 11x^2 \)
22. \( g(x) = 4x^3 - 5x^2 \)
23. \( h(x) = 6x^{1/2} + 3x \)
24. \( f(x) = x^{1/2} - x^{-1/2} \)
25. \( g(t) = \frac{2}{3t^2} \)
26. \( h(x) = \frac{10}{(7x)^2} \)
27. \( f(\theta) = 4\theta - 5\sin \theta \)
28. \( g(\alpha) = 4\cos \alpha + 6 \)
29. \( f(\theta) = 3\cos \theta - \frac{\sin \theta}{4} \)
30. \( g(\alpha) = \frac{5\sin \alpha}{3} - 2\alpha \)

Writing In Exercises 31 and 32, the figure shows the graphs of a
function and its derivative. Label the graphs as \( f \) or \( f' \) and
write a short paragraph stating the criteria you used in making
your selection. To print an enlarged copy of the graph, go to the
website \text{www.mathgraphs.com}.

31. \( \quad \)
32. \( \quad \)

33. Vibrating String When a guitar string is plucked, it vibrates
with a frequency of \( F = 200\sqrt{T} \), where \( F \) is measured in
vibrations per second and the tension \( T \) is measured in pounds.
Find the rate of change of \( F \) when \( a) T = 4 \) and \( b) T = 9 \).

34. Vertical Motion A ball is dropped from a height of 100 feet.
One second later, another ball is dropped from a height of
75 feet. Which ball hits the ground first?

35. Vertical Motion To estimate the height of a building, a weight
is dropped from the top of the building into a pool at ground
level. How high is the building if the splash is seen 9.2 seconds
after the weight is dropped?

36. Vertical Motion A bomb is dropped from an airplane at an alti-
itude of 14,400 feet. How long will it take for the bomb to reach
the ground? (Because of the motion of the plane, the fall will not
be vertical, but the time will be the same as that for a vertical
fall.) The plane is moving at 600 miles per hour. How far will the
bomb move horizontally after it is released from the plane?

37. Projectile Motion A thrown ball follows a path described by
\( y = x - 0.02x^2 \).
   (a) Sketch a graph of the path.
   (b) Find the total horizontal distance the ball is thrown.
   (c) At what \( x \)-value does the ball reach its maximum height
       (Use the symmetry of the path.)
   (d) Find an equation that gives the instantaneous rate of change
       of the height of the ball with respect to the horizontal
       change. Evaluate the equation at \( x = 0, 10, 25, 30, \) and \( 5 \)
   (e) What is the instantaneous rate of change of the height when
       the ball reaches its maximum height?
38. Projectile Motion The path of a projectile thrown at an angle of 45° with level ground is
\[ y = x - \frac{32}{v_0^2} (x^2) \]
where the initial velocity is \(v_0\) feet per second.
(a) Find the \(x\)-coordinate of the point where the projectile strikes the ground. Use the symmetry of the path of the projectile to locate the \(x\)-coordinate of the point where the projectile reaches its maximum height.
(b) What is the instantaneous rate of change of the height when the projectile is at its maximum height?
(c) Show that doubling the initial velocity of the projectile multiplies both the maximum height and the range by a factor of 4.
(d) Find the maximum height and range of a projectile thrown with an initial velocity of 70 feet per second. Use a graphing utility to graph the path of the projectile.

39. Horizontal Motion The position function of a particle moving along the \(x\)-axis is
\[ x(t) = t^2 - 3t + 2 \] for \(-\infty < t < \infty\).
(a) Find the velocity of the particle.
(b) Find the open \(t\)-interval(s) in which the particle is moving to the left.
(c) Find the position of the particle when the velocity is 0.
(d) Find the speed of the particle when the position is 0.

40. Modeling Data The speed of a car in miles per hour and the stopping distance in feet are recorded in the table.

<table>
<thead>
<tr>
<th>Speed</th>
<th>Stopping Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

(a) Use the regression capabilities of a graphing utility to find a quadratic model for the data.
(b) Use a graphing utility to plot the data and graph the model.
(c) Use a graphing utility to graph \(dy/dx\).
(d) Use the model to approximate the stopping distance at a speed of 65 miles per hour.
(e) Use the graphs in parts (b) and (c) to explain the change in stopping distance as the speed increases.

In Exercises 41–54, find the derivative of the function.

41. \(f(x) = (5x^2 + 8x)(x^2 - 4x - 6)\)
42. \(g(x) = (x^2 + 7x)(x + 3)\)
43. \(h(x) = \sqrt{x} \sin x\)
44. \(f(t) = 2t^3 \cos t\)
45. \(f(x) = \frac{x^2 + x - 1}{x^2 - 1}\)
46. \(f(x) = \frac{6x - 5}{x^2 + 1}\)
47. \(f(x) = \frac{1}{9 - 4x^2}\)
48. \(f(x) = \frac{9}{3x^2 - 2x}\)
49. \(y = \frac{x^4}{\cos x}\)
50. \(y = \frac{\sin x}{x^4}\)
51. \(y = 3x^2 \sec x\)
52. \(y = 2x - x^2 \tan x\)
53. \(y = x \cos x - \sin x\)
54. \(g(x) = 3x \sin x + x^2 \cos x\)

In Exercises 55–58, find an equation of the tangent line to the graph of \(f\) at the given point.

55. \(f(x) = \frac{2x^3 - 1}{x^2}, \quad (1, 1)\)
56. \(f(x) = \frac{x + 1}{x - 1}, \quad \left(\frac{1}{2}, -3\right)\)
57. \(f(x) = -x \tan x, \quad (0, 0)\)
58. \(f(x) = \frac{1 + \cos x}{1 - \cos x^2}, \quad \left(\frac{\pi}{2}, 1\right)\)

59. Acceleration The velocity of an object in meters per second is \(v(t) = 3t - t^2, \quad 0 \leq t \leq 6\). Find the velocity and acceleration of the object when \(t = 4\).

60. Acceleration The velocity of an automobile starting from rest is
\[ v(t) = \frac{90t}{4t + 10}, \]
where \(v\) is measured in feet per second. Find the vehicle’s velocity and acceleration at each of the following times.
(a) 1 second (b) 5 seconds (c) 10 seconds

In Exercises 61–66, find the second derivative of the function.

61. \(g(t) = -8t^3 - 5t + 12\)
62. \(h(x) = 2x^3 - 3x\)
63. \(f(x) = 15x^{12}\)
64. \(f(x) = 20 \sqrt{x}\)
65. \(f(t) = 3 \tan t\)
66. \(h(t) = 10 \cos t - 15 \sin t\)

In Exercises 67 and 68, show that the function satisfies the equation.

67. \(y = 2 \sin x + 3 \cos x\)
68. \(y = \frac{10 - \cos x}{x}\)

In Exercises 69–80, find the derivative of the function.

69. \(h(x) = \left(\frac{x + 5}{x^2 + 3}\right)^2\)
70. \(f(x) = \left(\frac{x^2 + 1}{x^2 + 3}\right)^3\)
71. \(f(x) = (x^2 - 1)^{3/2}(x^3 + 5)\)
72. \(h(t) = \frac{t}{(1 - t)^3}\)
73. \(y = 5 \cos(9x + 1)\)
74. \(y = 1 - \cos 2x + 2 \cos 2x\)
75. \(y = \frac{x - 2}{4} \sin 2x\)
76. \(y = \frac{x}{2} \sec^2 x\)
77. \(y = 2 \sin^{1/2} x - 2 \frac{\sin x}{\sin^{1/2} x}\)
78. \(f(x) = \frac{3x}{\sqrt{x^2 + 1}}\)
79. \(y = \frac{\sin mx}{x + 2}\)
80. \(y = \frac{\cos(x - 1)}{x - 1}\)

In Exercises 81–84, find the derivative of the function at the given point.

81. \(f(x) = \sqrt{1 - x^3}, \quad (-2, 3)\)
82. \(f(x) = \sqrt{x^2 - 1}, \quad (3, 2)\)
5. \( f(x) = (x-3)^{2/3} \)

Differentiable \((0,3),(3,\infty)\)

6. \( f(x) = \frac{3x}{x+1} \)

Differentiable \((0,-1),(1,\infty)\)

7. \( f(x) = 4 - |x-2| \)
   a) Yes; cont. at \(x=2\)
   b) No, sharp turn not diff at \(x=2\)

8. \( f(x) = \frac{1}{4}x^2 + x + 2 \)
   \(x < 2\)
   \(c^2 + 4c + 2 = 1 + 4(c-2) + (c-2)^2\)
   \(4 - 8 + 2 = 1 + 8 - 4\)
   \(-2 = 5\)
   a) No, not cont. at \(x=2\)
   b) No, jump at \(x=2\) not diff

9. \( g(x) = \frac{1}{3}x^2 - \frac{x}{6} \)
   \(\left(-1, \frac{5}{6}\right)\)

9. \( g(x) = \frac{1}{3}x - \frac{1}{6} \)

\( m = \frac{4}{3}(1) - \frac{1}{6} = \frac{4}{3} - \frac{1}{6} = \frac{8}{6} - \frac{1}{6} = \frac{7}{6} = \frac{7}{2} \)

\( y = \frac{5}{6} \)

\( y = \frac{2}{3}x - \frac{3}{2} \)

\( y = \frac{3}{2}x - \frac{9}{6} + \frac{5}{6} \)

\( y = \frac{3}{2}x - \frac{9}{6} + \frac{5}{6} \)

\( y = \frac{3}{2}x - \frac{9}{6} + \frac{5}{6} \)

11. \( f(x) = x^3 - 1, \ (-1,3) \)

\( f'(x) = 3x^2 \)

\( y' = 3x + 1 \)

12. \( f(x) = \sqrt{x} \sin x \)

\( \frac{x^{1/2}}{2} \sin x + \frac{1}{2} x^{1/2} \cos x \)

17. \( f(x) = x^6 \)

\( f'(x) = 6x^5 \)

19. \( h(t) = 13t^4 \)

\( h'(t) = 52t^3 \)

21. \( f(x) = x^2 - 1 \)

\( f'(x) = 3x^2 - 2 \)

23. \( h(x) = 6x^3 + 3x^2 \)

\( h'(x) = 18x^2 + 6x \)

25. \( g(t) = \frac{3}{3t^2} \)

\( g'(t) = -\frac{4}{3}t^{-3} \)

27. \( f(\theta) = 4\theta - 5\sin\theta \)

\( f'(\theta) = 4 - 5\cos\theta \)

29. \( f(\theta) = 3\cos\theta - \frac{\sin\theta}{4} \)

\( f'(\theta) = 3\cos\theta - \frac{\sin\theta}{4} \cos\theta \)

43. \( h(x) = \sqrt{x} \sin x \)

\( \frac{x^{1/2}}{2} \sin x + \frac{1}{2} x^{1/2} \cos x \)

45. \( f(x) = x^2 + x - 1 \)

\( \frac{x^2 + 1}{x - 1} \)

47. \( f(x) = \frac{1}{9 - 4x^2} \)

\( \frac{-8x}{(9 - 4x^2)^2} \)

49. \( y = \frac{x^4}{\cos x} \)

\( y' = \frac{4x^3 \cos x + x^4 \sin x}{\cos^2 x} \)

53. \( y = x \cos x - \sin x \)

\( y' = \cos x - x \sin x - \cos x \)

\( y' = \frac{x \cos x}{\sin x} \)

\( y' = \frac{x \cos x}{\sin x} - \cos x \)

\( y' = -x \sin x \)
55. \( f(x) = \frac{2x^3 - 1}{x^2} \); \((1, 1)\)
\[ f'(x) = 2 + 2x - \frac{2}{x^3} \]
\[ M = 2 + \frac{2}{1^3} = 4 \]
\[ y - 1 = 4(x - 1) \]
\[ y = 4x - 3 \]

56. \( f(x) = \frac{x + 1}{x - 1} \); \((\frac{1}{2}, -3)\)
\[ f'(x) = \frac{(x - 1) - (x + 1)}{(x - 1)^2} \]
\[ = \frac{-2}{(x - 1)^2} \]
\[ M = \frac{2}{(\frac{1}{2} - 1)^2} = \frac{2}{\frac{1}{4}} = -8 \]
\[ y + 3 = -8(x - \frac{1}{2}) \]
\[ y = -8x + 1 \]

61. \( g(t) = -(t^3 - 5t + 12) \)
\[ g'(t) = -3t^2 - 5 \]
\[ g''(t) = -6t \]

63. \( f(x) = 15x^5 + 2 \)
\[ f'(x) = \frac{75}{2}x^3 + \frac{2}{x} \]
\[ f''(x) = \frac{225}{4}x^2 \]

65. \( f(\theta) = 3 \tan \theta \)
\[ f'(\theta) = 3 \sec^2 \theta \]
\[ f''(\theta) = 6(\sec \theta) \cdot \sec \theta \cdot \tan \theta \]
\[ = 6 \sec^2 \theta \cdot \tan \theta \]

81. \( f(x) = \sqrt{1 - x^2} \); \((-3, 3)\)
\[ f'(x) = \frac{1}{2} \cdot (1 - x^2)^{-\frac{1}{2}} \cdot (-2x) \]
\[ = -\frac{3x^2}{\sqrt{1 - x^2}} \]

71. \( f(x) = (x^2 - 1)^{\frac{5}{2}} (x^3 + 5) \)
\[ f'(x) = 5x(x^2 - 1)^{\frac{3}{2}} (x^3 + 5) + 3x^2(x^2 - 1)^{\frac{1}{2}} \]

72. \( h(\theta) = \frac{\theta}{(1 - \theta)^3} \)
\[ h'(\theta) = \frac{(1 - \theta)^3 + 3\theta(1 - \theta)^2}{(1 - \theta)^6} \]
\[ = \frac{1 - \theta + 3\theta}{(1 - \theta)^4} \]
\[ = \frac{1 + 2\theta}{(1 - \theta)^4} \]

103. \( x^2 + 3xy + y^3 = 10 \)
\[ 2x + 3y + 3xy' + 3y^2 = 0 \]
\[ 3xy' + 3y^2y' = -2x - 3y \]
\[ y' = \frac{-2x - 3y}{3x + 3y^2} \]

112. \( \frac{ds}{dt} = 8 \text{ ft/sec} \)
\[ s = 6.5 \text{ cm} \]
Find \( \frac{dA}{dt} \)
\[ A = 6 \text{ ft}^2 \]
\[ \frac{dA}{dt} = 12 \frac{ds}{dt} \]
\[ \frac{dA}{dt} = 12(6.5)(8) \]
\[ \frac{dA}{dt} = 624 \text{ cm}^2/\text{sec} \]