

# FACTS EVERY GOOD MATHLETE SHOULD KNOW

1.) Quadratic Formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2.) Solving  $ax^2 + bx + c = 0$

- $b^2 - 4ac < 0 \Rightarrow$  no real roots
- $b^2 - 4ac = 0 \Rightarrow$  one double root
- $b^2 - 4ac > 0 \Rightarrow$  two real roots

Sum of roots is  $-\frac{b}{a}$

Product of roots is  $\frac{c}{a}$

Average of roots is  $-\frac{b}{2a}$

3.) First coordinate of vertex of parabola  $Y = ax^2 + bx + c$  is  $\frac{-b}{2a}$

4.) A.)  $\log_A C = T \Leftrightarrow A^T = C$

E.)  $\log A^B = B(\log A)$

B.)  $\log_A C = \frac{1}{\log_C A}$

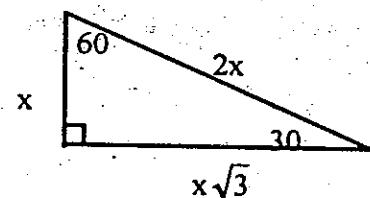
F.)  $\log \sqrt[B]{A^C} = \log A^{\frac{C}{B}} = \frac{C}{B}(\log A)$

C.)  $\log(A \cdot B) = \log A + \log B$

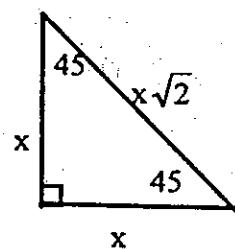
G.)  $\log_a b = \frac{\log_c b}{\log_c a}$

D.)  $\log\left(\frac{A}{B}\right) = \log A - \log B$

5.)  $30^\circ - 60^\circ -$  right triangle



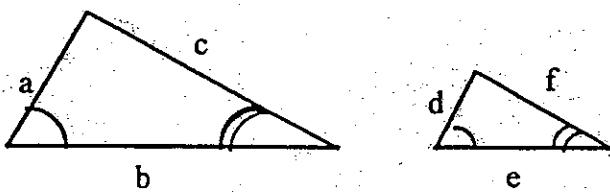
$45^\circ - 45^\circ -$  right triangle



Facts

Page 2

- 6.) Similar triangles:  $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$



- 7.) Slope:  $m = \frac{Y_2 - Y_1}{X_2 - X_1}$

Slope of  $Y = mx + b$  is  $m$  and  $Y$ -intercept is  $b$ .

Slope of  $AX + BY = C$  is  $-\frac{A}{B}$  and  $Y$ -intercept is  $\frac{C}{B}$ .

Slopes of parallel lines are equal if slopes are defined.

Slopes of perpendicular lines are negative reciprocals if slopes defined.

Slope of a vertical line is undefined.

- 8.) Arithmetic progression or sequence: Any two terms have same difference

$$T_1; T_1 + d; T_1 + 2d; T_1 + 3d; T_1 + 4d; \dots$$

any term:  $T_n = T_1 + (n - 1)d$

Sum of  $n$  terms:  $S_n = \frac{n}{2}[T_1 + T_n]$  or  $S_n = \frac{n}{2}[2T_1 + (n - 1)d]$

- 9.) Geometric progression or sequence: Any two terms have same ratio

$$T_1; T_1r; T_1r^2; T_1r^3; T_1r^4; \dots$$

Any term:  $T_n = T_1 \cdot r^{n-1}$

Sum of  $n$  terms:  $S_n = \frac{T_1(1-r^n)}{1-r}$  OR  $S_n = \frac{T_1 - r \cdot T_n}{1-r}$

Sum of infinite G.P.:  $S = \frac{T_1}{1-r}$  if  $|r| < 1$

- 10.) Arithmetic mean of  $a$  and  $b$  is  $\frac{a+b}{2}$

Geometric mean of  $a$  and  $b$  is  $\sqrt{a \cdot b}$

Facts

Page 3

- 11.) Number written in base  $r$

$$12345_r \Rightarrow 1 \cdot r^4 + 2 \cdot r^3 + 3 \cdot r^2 + 4r^1 + 5r^0$$

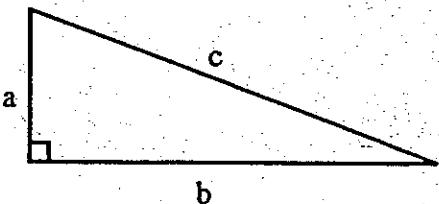
$$\text{i.e. } 123_4 = 1 \cdot 4^2 + 2 \cdot 4^1 + 3 \cdot 4^0 = 16 + 8 + 3 = 27$$

- 12.) Know method for changing repeating decimal to a fraction

$$\begin{array}{r} 9\overline{191} \\ 100N = 91.\overline{191} \\ N = \underline{.9191} \\ 99N = 91 \\ N = \frac{91}{99} \end{array}$$

- 13.) Pythagorean Theorem

$$a^2 + b^2 = c^2$$



Also midpoint of hypotenuse is equidistant from all three vertices

14.) Area:      Equilateral  $\Delta$ :       $A = \frac{s^2}{4}\sqrt{3}$       Regular Polygon:  $A = \frac{1}{2}a \cdot p$

Trapezoid:       $A = \frac{1}{2}h(b_1 + b_2)$        $a = \text{apothem}$

$p = \text{perimeter}$

Rhombus:       $A = \frac{1}{2}d_1 \cdot d_2$

Circle:       $A = \pi r^2$

Sphere:       $A = 4 \pi r^2$

Cone:       $A = \pi r^2 + 2 \pi r \ell$       ( $\ell$  is slant height)

Cylinder:       $A = 2 \pi r^2 + 2 \pi rh$

15.) Volume : Prism :  $V = Bh$        $B$  = Area of base

Cylinder :  $V = \pi r^2 h$

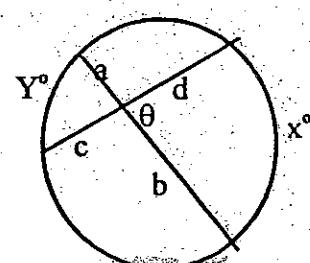
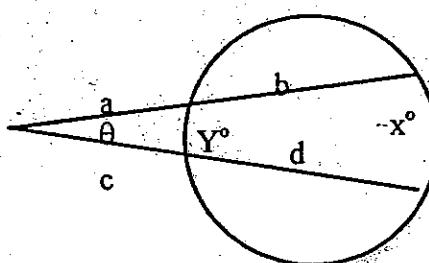
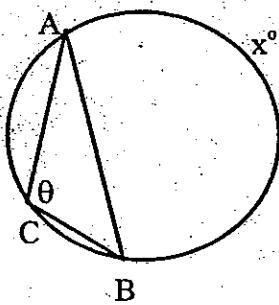
Pyramid :  $V = \frac{1}{3} B \cdot h$

Cone :  $V = \frac{1}{3} \pi r^2 h$

Sphere :  $V = \frac{4}{3} \pi r^3$

Cube :  $V = e^3$

16.) Circles



$$\theta = \frac{1}{2} \cdot$$

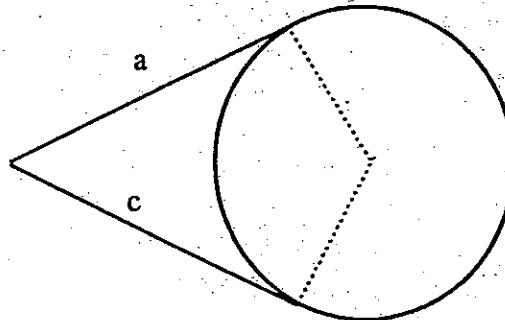
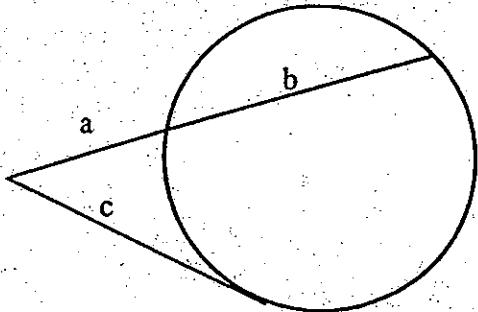
$$\theta = \frac{1}{2}(X - Y)$$

$$\theta = \frac{1}{2}(X + Y)$$

$\theta = 90^\circ$  if  $\overline{AB}$  is diameter

$$a(a + b) = c(c + d)$$

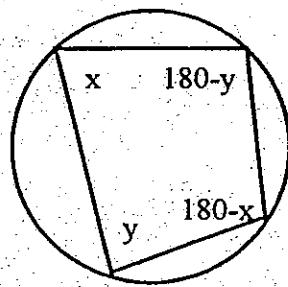
$$a \cdot b = c \cdot d$$



$$a(a + b) = c^2$$

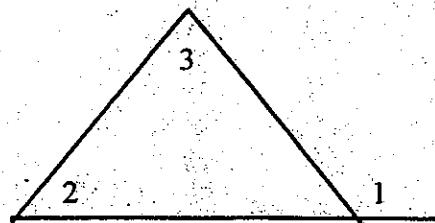
$$a = c$$

radius  $\perp$  tangent



Opposite angles of inscribed quadrilateral are supplementary

17.)



$$\angle 1 = \angle 2 + \angle 3$$

Exterior angle = Sum of remote interior angles

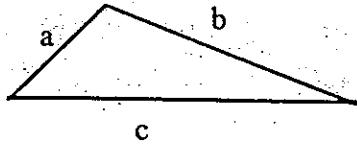
18.) Sum of interior angles of an  $n$ -gon is  $180(n - 2)$

Sum of exterior angles of an  $n$ -gon is  $360^\circ$

Each interior angle of a regular  $n$ -gon is  $\frac{180(n - 2)}{n}$

Each exterior angle of a regular  $n$ -gon is  $\frac{360}{n}$

19.)



1.) Sum of any 2 sides of triangle is greater than 3rd side

2.)  $|a - b| < c < a + b$

20.) A median is segment from vertex of triangle drawn to midpoint of opposite side

21.) Factoring:

$$x^2 - y^2 = (x - y)(x + y)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$x^2 + y^2$  is prime

Facts

Page 6

- 22.) The area of a convex  $n$ -gon with vertices  $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots, (x_n, y_n)$  is

$$A = \frac{1}{2} \left| \begin{array}{c|c} x_1 & y_1 \\ \hline x_2 & y_2 \end{array} \right| + \left| \begin{array}{c|c} x_2 & y_2 \\ \hline x_3 & y_3 \end{array} \right| + \left| \begin{array}{c|c} x_3 & y_3 \\ \hline x_4 & y_4 \end{array} \right| + \dots + \left| \begin{array}{c|c} x_n & y_n \\ \hline x_1 & y_1 \end{array} \right|$$

Absolute value symbol      Determinant symbol

- 23.) In similar polygons:  $\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$

In similar solids:  $\frac{V_1}{V_2} = \left(\frac{S_1}{S_2}\right)^3$

- 24.) The numbers 1 and 51 are not prime!

- 25.) If  $X^Y = 1$ , then  $X \neq 0$  and  $Y = 0$  or  $X = 1$  and  $Y = R$  or  $X = -1$  and  $Y = \text{even integer}$

- 26.) The number of permutations of  $n$  different objects is  $n!$ . That is  ${}_n P_n = n!$

- 27.) The number of permutations of  $r$  objects selected from  $n$  different objects is  $\frac{n!}{(n-r)!}$ ; that is

$${}_n P_r = \frac{n!}{(n-r)!}$$

- 28.) The number of permutations of  $n$  objects, not all different,  $a$  of one type,  $b$  of another type,  $c$  of a 3rd type, etc. is  $\frac{n!}{a!b!c!...}$

- 29.) The number of combinations of  $r$  objects selected from  $n$  different objects is  $\frac{n!}{(n-r)!r!}$ ; that is

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

- 30.) Know difference between mean, median, and mode.

31.)  $\cos(a \pm B) = \cos a \cos B \pm \sin a \sin B$

$\sin(a \pm B) = \sin a \cos B \pm \cos a \sin B$