12-1 Lines That Intersect Circles

[Warm Up]

[Lesson Presentation]

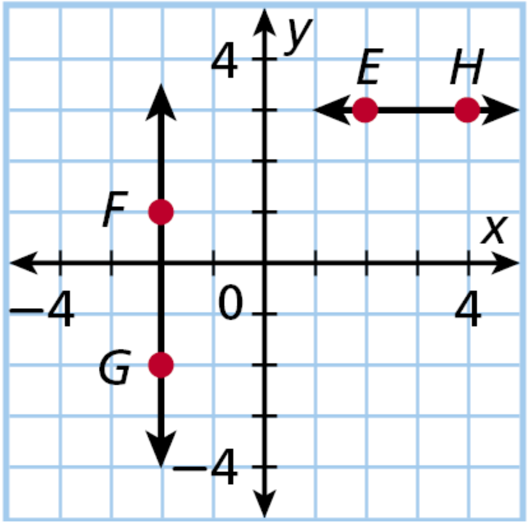
[Lesson Quiz]

Warm Up

Write the equation of each item.

1. $\overleftrightarrow{FG}$ $x = -2$

2. $\overleftrightarrow{EH}$ $y = 3$



3. $2(25 - x) = x + 2$

$x = 16$

4. $3x + 8 = 4x$

$x = 8$

Objectives

Identify tangents, secants, and chords.

Use properties of tangents to solve problems.

Vocabulary

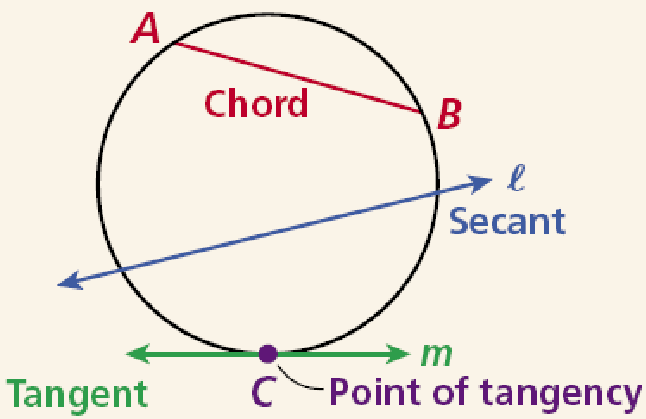
interior of a circle	concentric circles
exterior of a circle	tangent circles
chord	common tangent
secant	
tangent of a circle	
point of tangency	
congruent circles	

This photograph was taken 216 miles above Earth. From this altitude, it is easy to see the curvature of the horizon. Facts about circles can help us understand details about Earth.

Recall that a circle is the set of all points in a plane that are equidistant from a given point, called the center of the circle. A circle with center *C* is called circle *C*, or $\odot C$.

The **interior of a circle** is the set of all points inside the circle. The **exterior of a circle** is the set of all points outside the circle.

Lines and Segments That Intersect Circles

TERM	DIAGRAM
A **chord** is a segment whose endpoints lie on a circle.	
A **secant** is a line that intersects a circle at two points.	
A **tangent** is a line in the same plane as a circle that intersects it at exactly one point.	
The point where the tangent and a circle intersect is called the **point of tangency**.	

Example 1: Identifying Lines and Segments That Intersect Circles

Identify each line or segment that intersects ⊙*L*.

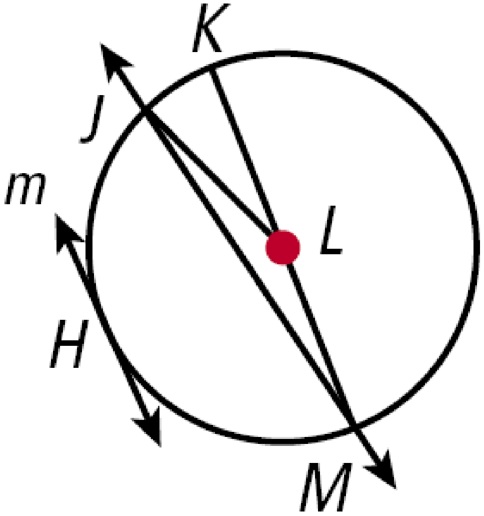
chords: $\overline{JM}$ and $\overline{KM}$

secant: $\overleftrightarrow{JM}$

tangent: m

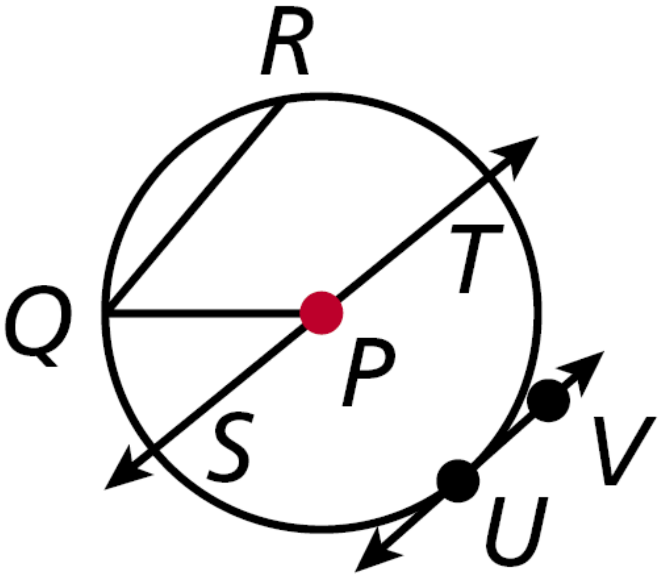
diameter: $\overline{KM}$

radii: $\overline{LK}$, $\overline{LJ}$, and $\overline{LM}$



Check It Out! Example 1

Identify each line or segment that intersects ⊙*P*.



chords: $\overline{QR}$ and $\overline{ST}$

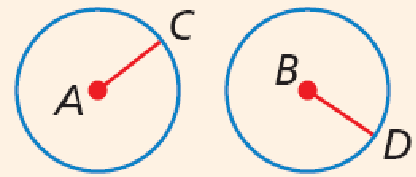
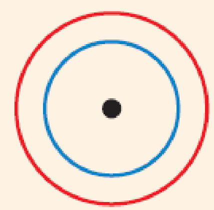
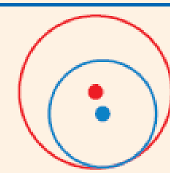
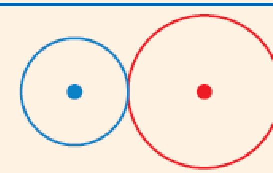
secant: $\overleftrightarrow{ST}$

tangent: $\overleftrightarrow{UV}$

diameter: $\overline{ST}$

radii: $\overline{PQ}$, $\overline{PT}$, and $\overline{PS}$

Pairs of Circles

TERM	DIAGRAM
Two circles are **congruent circles** if and only if they have congruent radii.	 $\odot A \cong \odot B$ *if* $\overline{AC} \cong \overline{BD}$. $\overline{AC} \cong \overline{BD}$ *if* $\odot A \cong \odot B$.
Concentric circles are coplanar circles with the same center.	
Two coplanar circles that intersect at exactly one point are called **tangent circles**.	 Internally tangent circles  Externally tangent circles

Example 2: Identifying Tangents of Circles

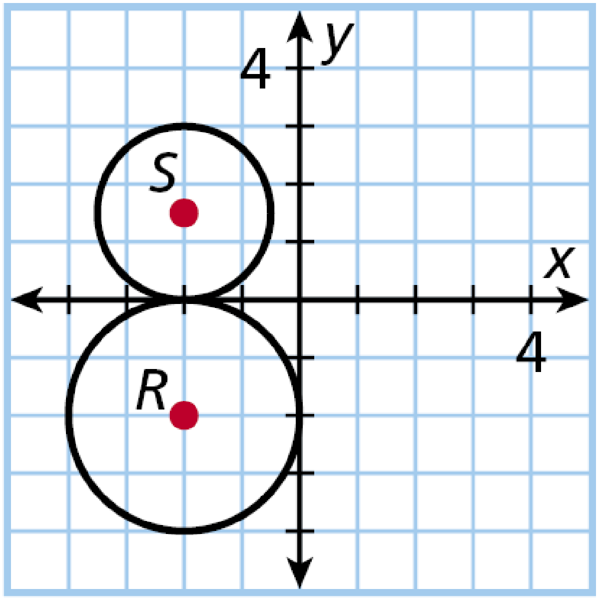
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

radius of ⊙*R*: 2

Center is (–2, –2). Point on ⊙ is (–2,0). Distance between the 2 points is 2.

radius of ⊙*S*: 1.5

Center is (–2, 1.5). Point on ⊙ is (–2,0). Distance between the 2 points is 1.5.



Example 2 Continued

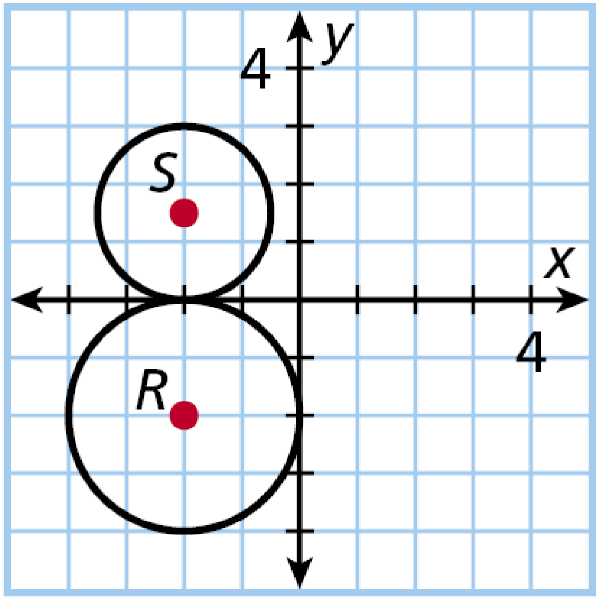
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

point of tangency: (–2, 0)

Point where the ⊙s and tangent line intersect

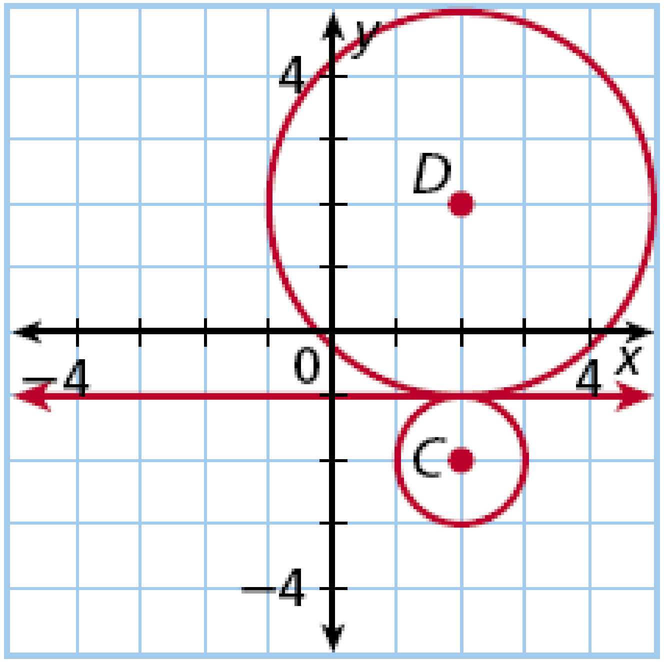
equation of tangent line: $y = 0$

Horizontal line through (–2,0)



Check It Out! Example 2

Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.



radius of ⊙*C*: 1

Center is (2, –2). Point on ⊙ is (2, –1). Distance between the 2 points is 1.

radius of ⊙*D*: 3

Center is (2, 2). Point on ⊙ is (2, –1). Distance between the 2 points is 3.

Check It Out! Example 2 Continued

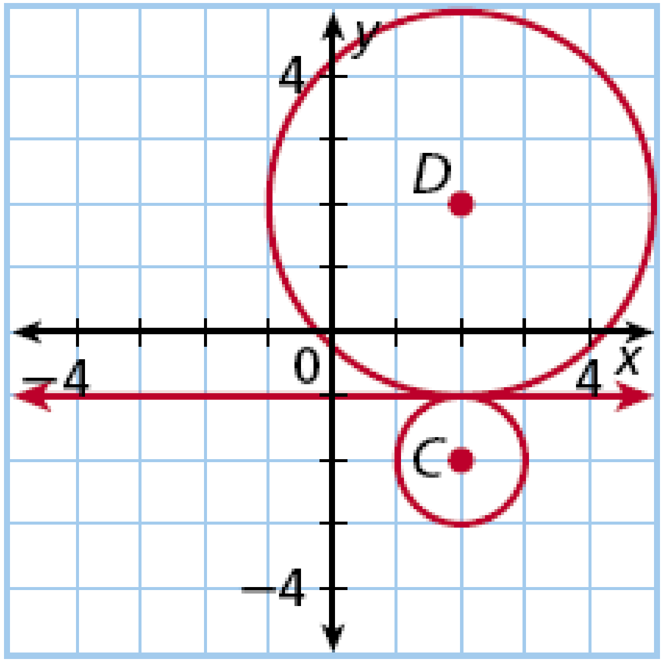
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

Pt. of tangency: **(**2, –1**)**

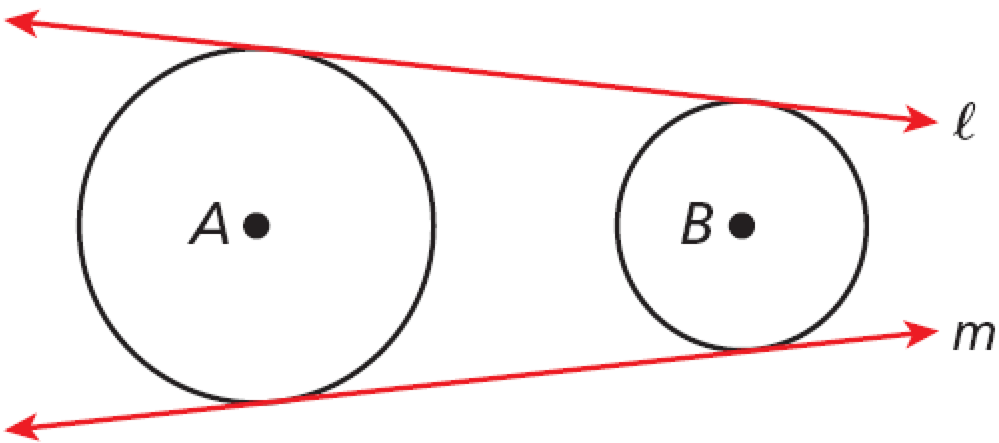
Point where the ⊙s and tangent line intersect

eqn. of tangent line: $y = -1$

Horizontal line through (2,-1)

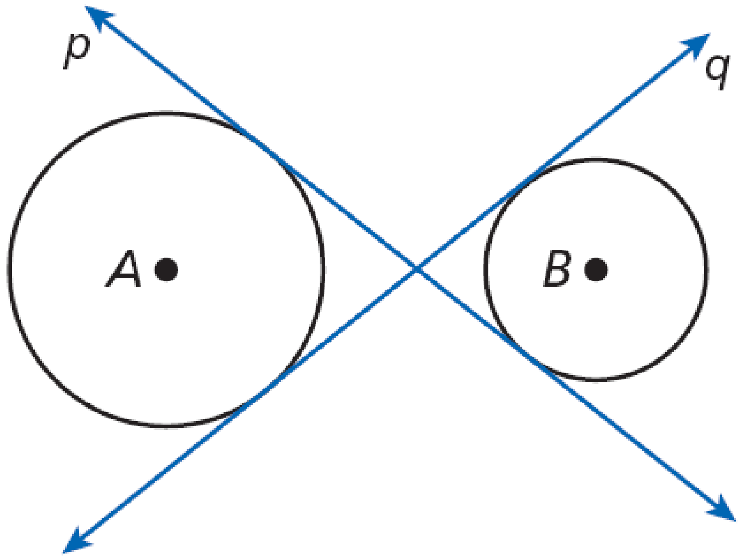


A **common tangent** is a line that is tangent to two circles.



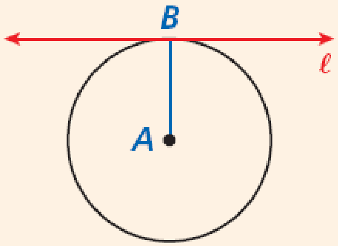
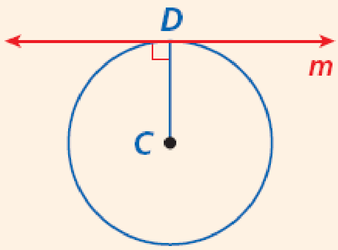
Lines ℓ and m are common external tangents to $\odot A$ and $\odot B$.

A **common tangent** is a line that is tangent to two circles.



Lines p and q are common internal tangents to $\odot A$ and $\odot B$.

Theorems

THEOREM	HYPOTHESIS	CONCLUSION
11-1-1 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. (line tangent to ⊙ → line ⊥ to radius)	 ℓ is tangent to ⊙*A*	$\ell \perp \overline{AB}$
11-1-2 If a line is perpendicular to a radius of a circle at a point on the circle, then the line is tangent to the circle. (line ⊥ to radius → line tangent to ⊙)	 *m* is ⊥ to $\overline{CD}$ at *D*	*m* is tangent to ⊙*C*.

Example 3: Problem Solving Application

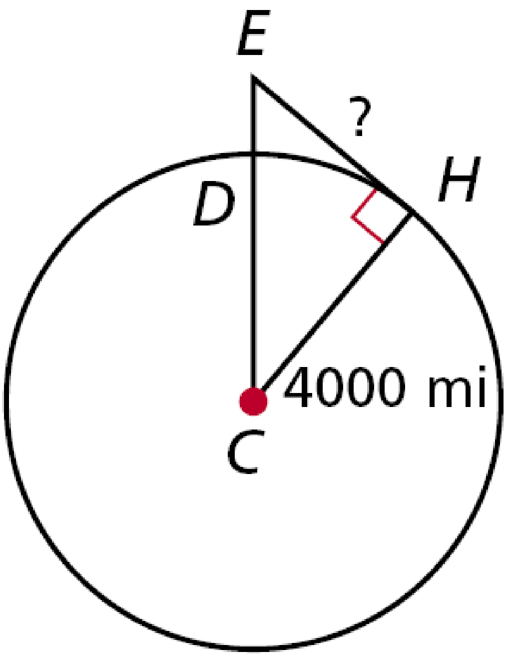
Early in its flight, the Apollo 11 spacecraft orbited Earth at an altitude of 120 miles. What was the distance from the spacecraft to Earth's horizon rounded to the nearest mile?

1 Understand the Problem

The **answer** will be the length of an imaginary segment from the spacecraft to Earth's horizon.

2 Make a Plan

Draw a sketch. Let *C* be the center of Earth, *E* be the spacecraft, and *H* be a point on the horizon. You need to find the length of $\overline{EH}$, which is tangent to ⊙*C* at *H*. By Theorem 11-1-1, $\overline{EH} \perp \overline{CH}$. So $\triangle CHE$ is a right triangle.



3 Solve

$EC = CD + ED$ — *Seg. Add. Post.*

$= 4000 + 120 = 4120$ mi — *Substitute 4000 for CD and 120 for ED.*

$EC^2 = EH^2 + CH^2$ — *Pyth. Thm.*

$4120^2 = EH^2 + 4000^2$ — *Substitute the given values.*

$974{,}400 = EH^2$ — *Subtract 4000^2 from both sides.*

$987 \text{ mi} \approx EH$ — *Take the square root of both sides.*

4 Look Back

The problem asks for the distance to the nearest mile. Check if your answer is reasonable by using the Pythagorean Theorem. Is $987^2 + 4000^2 \approx 4120^2$?

Yes, $16{,}974{,}169 \approx 16{,}974{,}400$.

Check It Out! Example 3

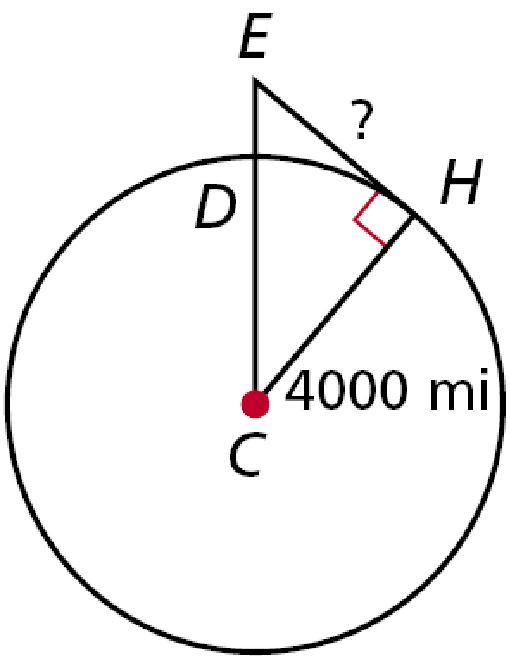
Kilimanjaro, the tallest mountain in Africa, is 19,340 ft tall. What is the distance from the summit of Kilimanjaro to the horizon to the nearest mile?

1 Understand the Problem

The **answer** will be the length of an imaginary segment from the summit of Kilimanjaro to the Earth's horizon.

2 Make a Plan

Draw a sketch. Let *C* be the center of Earth, *E* be the summit of Kilimanjaro, and *H* be a point on the horizon. You need to find the length of $\overline{EH}$, which is tangent to ⊙*C* at *H*. By Theorem 11-1-1, $\overline{EH} \perp \overline{CH}$. So Δ*CHE* is a right triangle.



3 Solve

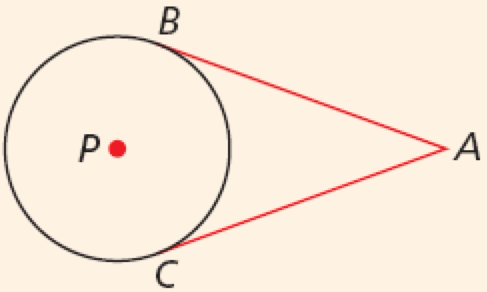
$ED = 19{,}340$	*Given*
$= \dfrac{19{,}340}{5{,}280} \approx 3.66$ mi	*Change ft to mi.*
$EC = CD + ED$	*Seg. Add. Post.*
$= 4000 + 3.66$	*Substitute 4000 for CD and 3.66 for ED.*
$= 4003.66$mi	
$EC^2 = EH^2 + CH^2$	*Pyth. Thm.*
$4003.66^2 = EH^2 + 4000^2$	*Substitute the given values.*
$29{,}293 = EH^2$	*Subtract 4000^2 from both sides.*
$171 \approx EH$	*Take the square root of both sides.*

4 Look Back

The problem asks for the distance from the summit of Kilimanjaro to the horizon to the nearest mile. Check if your answer is reasonable by using the Pythagorean Theorem. Is $171^2 + 4000^2 \approx 4004^2$?

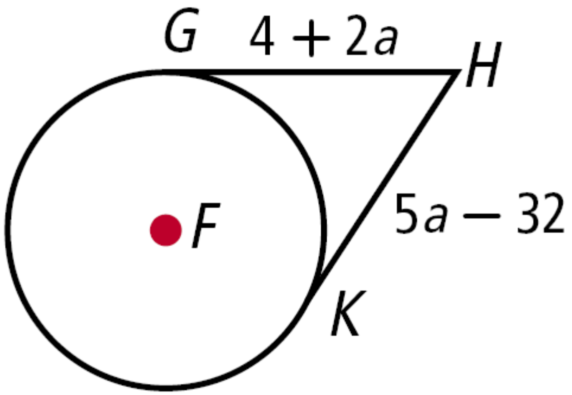
Yes, $16{,}029{,}241 \approx 16{,}032{,}016$.

Theorem 11-1-3

THEOREM	HYPOTHESIS	CONCLUSION
If two segments are tangent to a circle from the same external point, then the segments are congruent. (2 segs. tangent to ⊙ from same ext. pt. → segs. ≅)	 $\overline{AB}$ and $\overline{AC}$ are tangent to ⊙P.	$\overline{AB} \cong \overline{AC}$

Example 4: Using Properties of Tangents

$\overline{HK}$ and $\overline{HG}$ are tangent to ⊙F. Find HG.



$HK = HG$ — 2 segments tangent to ⊙ from same ext. point → segments ≅.

$5a - 32 = 4 + 2a$ — Substitute 5a – 32 for HK and 4 + 2a for HG.

$3a - 32 = 4$ — Subtract 2a from both sides.

$3a = 36$ — Add 32 to both sides.

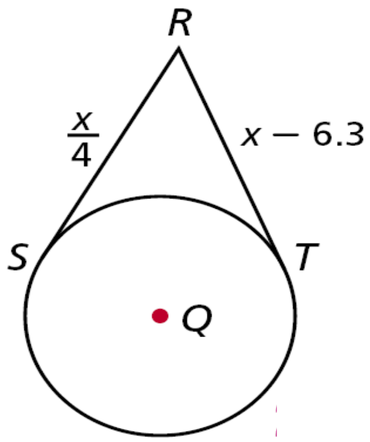
$a = 12$ — Divide both sides by 3.

$HG = 4 + 2(12)$ — Substitute 12 for a.

$= 28$ — Simplify.

Check It Out! Example 4a

$\overline{RS}$ and $\overline{RT}$ are tangent to ⊙Q. Find RS.



$RS = RT$ — *2 segments tangent to ⊙ from same ext. point → segments ≅.*

$\frac{x}{4} = x - 6.3$ — *Substitute $\frac{x}{4}$ for RS and $x - 6.3$ for RT.*

$x = 4x - 25.2$ — *Multiply both sides by 4.*

$-3x = -25.2$ — *Subtract 4x from both sides.*

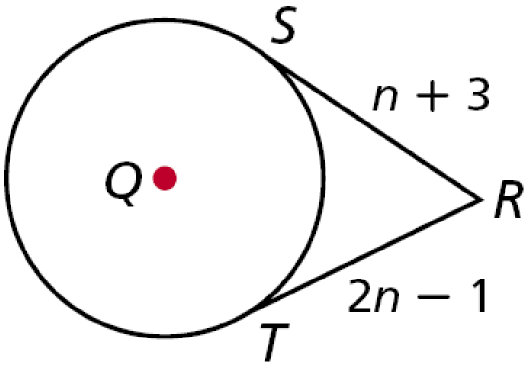
$x = 8.4$ — *Divide both sides by –3.*

$RS = \frac{8.4}{4}$ — *Substitute 8.4 for x.*

$= 2.1$ — *Simplify.*

Check It Out! Example 4b

$\overline{RS}$ and $\overline{RT}$ are tangent to ⊙Q. Find RS.



$RS = RT$ — *2 segments tangent to ⊙ from same ext. point → segments ≅.*

$n + 3 = 2n - 1$ — *Substitute n + 3 for RS and 2n – 1 for RT.*

$4 = n$ — *Simplify.*

$RS = 4 + 3$ — *Substitute 4 for n.*

$= 7$ — *Simplify.*

Lesson Quiz: Part I

1. Identify each line or segment that intersects $\odot Q$.

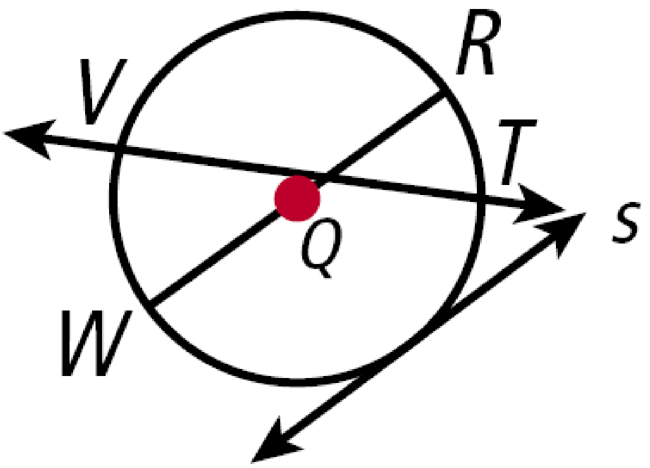
chords $\overline{VT}$ and $\overline{WR}$

secant: $\overleftrightarrow{VT}$

tangent: s

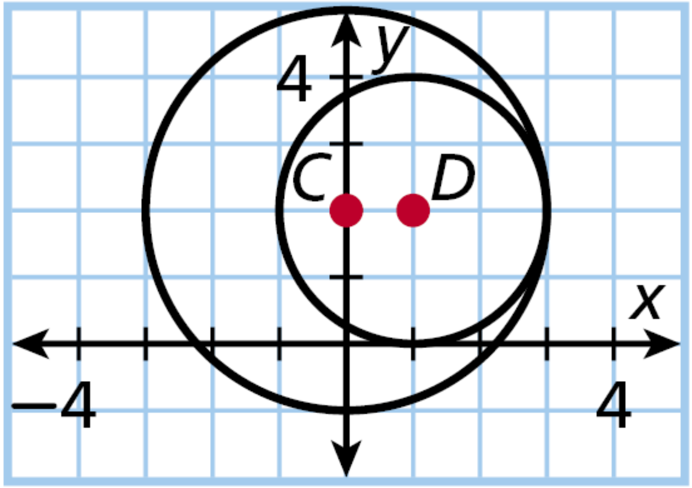
diam.: $\overline{WR}$

radii: $\overline{QW}$ and $\overline{QR}$



Lesson Quiz: Part II

2. Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.



radius of ⊙C: 3

radius of ⊙D: 2

pt. of tangency: **(**3, 2**)**

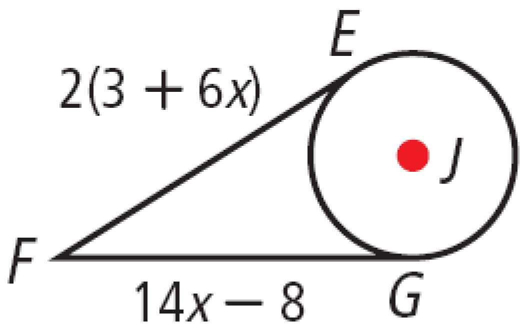
eqn. of tangent line: $x = 3$

Lesson Quiz: Part III

3. Mount Mitchell peaks at 6,684 feet. What is the distance from this peak to the horizon, rounded to the nearest mile?

$\approx$ 101 mi

4. $\overline{FE}$ and $\overline{FG}$ are tangent to $\odot F$. Find FG.



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